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John Robert Betten
Iowa State University

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A FREQUENCY DOMAIN AND s-PLANE ANALYSIS OF REFLECTING OBJECTS

by

John Robert Betten

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Head of Major Department

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Dean of Graduate College

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I. INTRODUCTION

A. Purpose

The purpose of this analysis is to investigate the frequency domain characteristics and s-plane characteristics of reflecting objects and to indicate the basic information available in these domains which should prove useful in the representation and identification of such objects. The investigation is particularly concerned with reflecting objects which are of interest in the radio and radar portion of the frequency spectrum. It is intended that the results of this study will be used as a guide for later experimental efforts, but the experiments themselves are beyond the intended scope of this study.

B. Preliminary Remarks

1. Signal representation

Since the signals (both transmitted and received) to be discussed in this analysis can be completely described in either the time or frequency domain, the discussion will frequently switch from one domain representation to the other, the choice depending upon the domain which offers the clearer presentation. The transformations which will be used to switch back and forth between the two domains are the direct Fourier transformation (1, p. 100),

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\, dt,$$

and the inverse Fourier transformation (1, p. 100),

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{+j\omega t} \, d\omega \; .$$

Although the signals, $f(t)$, under analysis will be real functions of the real variable, t, the functions, $F(\omega)$, will be complex functions of the real variable, ω. Another important transformation which will be used is the direct Laplace transformation (1, p. 104),

$$G(s) = \int_{o^+}^{\infty} f(t) e^{-st} \, dt \qquad (s = \sigma + j\omega) \; ,$$

which transforms the real function, $f(t)$, of the real variable, t, into a complex function, $G(s)$, of the complex variable, s. It is interesting to note that if the real part, σ, of the complex variable, s, is taken to be zero, the s-plane is effectively replaced by the imaginary axis, and $G(s)$ becomes $G(j\omega)$. It should also be noted that $G(j\omega)$ is identical to $F(\omega)$ providing that $f(t)$ is zero for $t < o$. The signals of interest in this discussion will be specified to be zero for $t < o$. Thus,

$$F(\omega) \equiv G(j\omega) \; .$$

To transform from the s-plane back to the time domain, the inverse Laplace transformation (1, p. 105) is used, i.e.,

$$f(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(s) e^{+st} \, ds \; .$$

2. <u>The transfer function concept</u>

The preceding comments with regard to signal representation are rather standard in discussions regarding electrical networks. It will also be useful to utilize the so-called transfer-function concept which also comes from the theory of networks. To introduce this concept, let a linear passive electrical network be excited by an input voltage, $e_{in}(t)$, having a Fourier transform, $E_{in}(\omega)$. Also let the output voltage be $e_{out}(t)$ with Fourier transform, $E_{out}(\omega)$. The ratio of the transformed output to the transformed input is defined as the network transfer function, (4, p. 222) $F_t(\omega)$. Thus,

$$F_t(\omega) = \frac{E_{out}(\omega)}{E_{in}(\omega)} = \frac{\int_{-\infty}^{+\infty} e_{out}(t)e^{-j\omega t}\,dt}{\int_{-\infty}^{+\infty} e_{in}(t)e^{-j\omega t}\,dt} .$$

It can be noted immediately at this point that

$$F_t(\omega) = E_{out}(\omega) \Big|_{\text{when } e_{in}(t) = \delta(t)} ,$$

since the Fourier Transform of the unit impulse, $\delta(t)$, is unity as shown in Appendix A. Therefore the system transfer-function is equal to the Fourier transform of the network output when the network is excited with a unit impulse. The output itself, (with a unit impulse as input) is generally referred to as the "impulse response" or "impulsive response" (4, p. 223) of the network. Clearly then, the impulsive response and the transfer function form a Fourier-transform pair.

When using the direct Laplace transformation, a slightly different expression is used for the network transfer-function (9, p. 157), i.e.,

$$G_t(s) = \frac{E_{out}(s)}{E_{in}(s)} = \frac{\int_{0^+}^{\infty} e_{out}(t)e^{-st}\,dt}{\int_{0^+}^{\infty} e_{in}(t)e^{-st}\,dt} \quad .$$

Since all the input and output functions of interest in this discussion are zero for $t < o$, a simple relation exists between the two transfer functions, namely, $F_t(\omega) \equiv G_t(j\omega)$. Also, since the Laplace transform of a unit impulse is unity as shown in Appendix A, it follows that

$$G_t(s) = E_{out}(s) \Big|_{\text{when } e_{in}(t) = \delta(t).}$$

Clearly, then, $G_t(s)$ and the impulsive response of the network form a Laplace transform pair. Hereafter the impulsive response of a network will be symbolized by $y(t)$, i.e.,

$$y(t) \triangleq e_{out}(t) \Big|_{\text{when } e_{in}(t) = \delta(t).}$$

$$= L^{-1} \left\{G_t(s)\right\}$$

$$= F^{-1} \left\{F_t(\omega)\right\}$$

The advantage of the transfer function concept lies in the ability to write

$$E_{out}(s) = E_{in}(s)G_t(s)$$

or

$$E_{out}(\omega) = E_{in}(\omega)F_t(\omega)$$

This means that once the transfer function is found either by analytical or experimental techniques, it can be used with any arbitrary $E_{in}(s)$ or $E_{in}(\omega)$ to determine the corresponding $E_{out}(s)$ or $E_{out}(\omega)$, respectively.

C. Network Response to a Modulated Carrier

Having recalled these few elementary notions from the theory of networks, let attention now be focused on the following problem: Consider a single sinusoidal carrier signal, $c(t)$, given by

$$c(t) = \text{Cos}\ (\omega_c t).$$

Let this signal, $c(t)$, be passed through an amplitude-modulating system which produces an output, $s(t)$, given by

$$s(t) = m(t)\text{Cos}(\omega_c t).$$

Consider further that the function, $m(t)$, is Laplace transformable with transform, $M(s)$. Under this hypothesis the signal, $s(t)$, is also Laplace transformable. Since multiplication in the time domain gives rise to convolution in the s domain, the transform, $S(s)$, of $s(t)$ is given by (1, p. 275):

$$S(s) = M(s) \circledast C(s) = \frac{1}{2\pi j} \int_{d_2-j\infty}^{d_2+j\infty} M(s-\omega)C(\omega)d\omega\ ,$$

where the symbol, $\circledast$, is used to denote the convolution operation. Although this may appear to be a rather formidable expression, the evaluation of S(s) can be performed rather simply due to the simple form of c(t). Rather than evaluating the convolution integral, the expression for s(t) is written with c(t) replaced by its complex exponential form. Thus

$$s(t) = m(t) \left[\frac{e^{+j\omega_c t} + e^{-j\omega_c t}}{2}\right] \text{, and}$$

$$S(s) = \int_{o^+}^{\infty} m(t) \left[\frac{e^{+j\omega_c t} + e^{-j\omega_c t}}{2}\right] e^{-st}\, dt$$

$$= \frac{1}{2}\int_{o^+}^{\infty} m(t) \left[e^{-[s-j\omega_c]t}\right] dt + \frac{1}{2}\int_{o^+}^{\infty} m(t) \left[e^{-[s+j\omega_c]t}\right] dt$$

$$= \frac{1}{2}\left[M(s-j\omega_c) + M(s+j\omega_c)\right] .$$

Thus the Laplace transform of S(s) is simply expressed.

Consider now that the signal, s(t), is applied to a linear passive time invariant network possessing a transfer function, $G_t(s)$. Let the output of this network be called r(t) with Laplace transform, R(s). Then

$$R(s) = G_t(s)S(s)$$

$$= \frac{G_t(s)}{2}\left[M(s-j\omega_c) + M(s+j\omega_c)\right] .$$

Now, $r(t) = L^{-1}[R(s)] = \frac{1}{2} L^{-1}\left[G_t(s)M(s-j\omega_c) + G_t(s)M(s+j\omega_c)\right]$

thus

$$r(t) = \frac{1}{2}\left\{L^{-1}\left[G_t(s)M(s-j\omega_c)\right] + L^{-1}\left[G_t(s)M(s+j\omega_c)\right]\right\} .$$

Now according to a basic theorem (1, p. 245) in Laplace transform theory, $r(t)$ can be written

$$r(t) = \frac{1}{2}\left\{L^{-1}[G_t(s+j\omega_c)M(s)]e^{+j\omega_c t} + L^{-1}[G_t(s-j\omega_c)M(s)]e^{-j\omega_c t}\right\} .$$

Now let $G_t(s+j\omega_c) = H(s)$ and let $G_t(s-j\omega_c) = F(s)$.

Then $L^{-1}[G(s+j\omega_c)M(s)] = L^{-1}[H(s)M(s)] = L^{-1}[M(s)H(s)]$,

and

$$L^{-1}[G(s-j\omega_c)M(s)] = L^{-1}[F(s)M(s)] = L^{-1}[M(s)F(s)] .$$

However,

$$L^{-1}[M(s)H(s)] = \int_0^t m(t-\tau)h(\tau)d\tau,$$

where $h(\tau) = L^{-1}[H(s)] = L^{-1}[G_t(s+j\omega_c)] = g(\tau)e^{-j\omega_c\tau}$

and $f(\tau) = L^{-1}[F(s)] = L^{-1}[G_t(s-j\omega_c)] = g(\tau)e^{+j\omega_c\tau}$

Thus,

$$L^{-1}[G_t(s+j\omega_c)M(s)] = \int_0^t m(t-\tau)g(\tau)e^{-j\omega_c\tau}\,d\tau ,$$

and,

$$L^{-1}[G_t(s-j\omega_c)M(s)] = \int_0^t m(t-\tau)g(\tau)e^{+j\omega_c\tau}\,d\tau .$$

However,

$$e^{-j\omega_c\tau} = \text{Cos } \omega_c\tau - j \text{ Sin } \omega_c\tau ,$$

and $e^{+j\omega_c\tau} = \text{Cos } \omega_c\tau + j \text{ Sin } \omega_c\tau$.

Consequently,

$$L^{-1}[G_t(s+j\omega_c)M(s)] = \int_0^t m(t-\tau)g(\tau)\text{Cos}(\omega_c\tau)d\tau - j\int_0^t m(t-\tau)g(\tau)\text{Sin}(\omega_c\tau)d\tau$$

and

$$L^{-1}[G_t(s-j\omega_c)M(s)] = \int_0^t m(t-\tau)g(\tau)\text{Cos}(\omega_c\tau)d\tau + j\int_0^t m(t-\tau)g(\tau)\text{Sin}(\omega_c\tau)d\tau.$$

Now, let $\int_0^t m(t-\tau)g(\tau) \text{ Cos}(\omega_c\tau)d\tau = p_{\omega_c}(t)$,

and let $-\int_0^t m(t-\tau)g(\tau)\text{Sin}(\omega_c\tau)d\tau = q_{\omega_c}(t)$.

Then,

$$L^{-1}[G_t(s+j\omega_c)M(s)] = p_{\omega_c}(t) + jq_{\omega_c}(t)$$

and $L^{-1}[G_t(s-j\omega_c)M(s)] = p_{\omega_c}(t) - jq_{\omega_c}(t)$.

Consequently, the previously established equation

$$r(t) = \frac{1}{2}\left\{L^{-1}[G_t(s+j\omega_c)M(s)]e^{+j\omega_c t} + L^{-1}[G_t(s-j\omega_c)M(s)]e^{-j\omega_c t}\right\},$$

can be written as

$$r(t) = \frac{1}{2}\left\{[p_{\omega_c}(t) + jq_{\omega_c}(t)]e^{+j\omega_c t} + [p_{\omega_c}(t) - jq_{\omega_c}(t)]e^{-j\omega_c t}\right\}$$

$$= \frac{1}{2}\left\{[jq_{\omega_c}(t)e^{j\omega_c t} - jq_{\omega_c}(t)e^{-j\omega_c t}] + [p_{\omega_c}(t)e^{j\omega_c t} + p_{\omega_c}(t)e^{-j\omega_c t}]\right\}$$

$$\text{or, } r(t) = -q_{\omega_c}(t)\left[\frac{e^{j\omega_c t} - e^{-j\omega_c t}}{2j}\right] + p_{\omega_c}(t)\left[\frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2}\right]$$

$$= p_{\omega_c}(t) \operatorname{Cos} \omega_c t - q_{\omega_c}(t) \operatorname{Sin} \omega_c t$$

$$= \operatorname{Re}\left\{[p_{\omega_c}(t) + jq_{\omega_c}(t)] \quad [\operatorname{Cos} \omega_c t + j \operatorname{Sin} \omega_c t]\right\}$$

$$= \operatorname{Re}\left\{[p_{\omega_c}(t) + jq_{\omega_c}(t)] \, e^{j\omega_c t}\right\} = \operatorname{Re}\left\{a_{\omega_c}(t)e^{j\phi(t)} e^{j\omega_c t}\right\}$$

$$= \operatorname{Re}\left\{a_{\omega_c}(t)e^{j[\omega_c t + \phi(t)]}\right\} = a_{\omega_c}(t) \operatorname{Cos} [\omega_c t + \phi_{\omega_c}(t)] .$$

Thus

$$r(t) = a_{\omega_c}(t) \operatorname{Cos} [\omega_c t + \phi_{\omega_c}(t)] .$$

where $a_{\omega_c}(t) = \sqrt{[p_{\omega_c}(t)]^2 + [q_{\omega_c}(t)]^2}$

and $\phi_{\omega_c}(t) = \tan^{-1} \left[\frac{q_{\omega_c}(t)}{p_{\omega_c}(t)}\right]$.

It is thus observed that the final form for the network output signal, $r(t)$, is a completely general expression for a modulated sine wave in that it allows for both amplitude modulation and phase modulation. Note also that the amplitude-modulating function and the phase-modulating function are dependent upon the value of the carrier ω_c, i.e., the position that the spectral carrier occupies in the ω spectrum. This dependence is symbolized by the use of the ω_c subscripts.

D. Reflecting Object Response to a Modulated Carrier

Now, consider a somewhat similar situation in which a transmitting radar is used to illuminate a reflecting object and a radar receiver is used to collect the reflected return. Let the transmitted signal, s(t), be an amplitude-modulated sinusoidal carrier and let the transmitter be initially turned on at the time t = o. Then s(t) can be expressed as:

$$s(t) = m(t) \operatorname{Cos} \omega_c t \qquad t \geq o$$
$$= o \qquad t < o ,$$

where m(t) is the amplitude modulating function and ω_c is the angular velocity of the sinusoidal carrier. This form of s(t) is sufficiently general to permit c-w (continuous-wave) operation since m(t) can be a step function or an extremely wide (long-duration) pulse. The transmitted signal is represented in the frequency domain by its amplitude spectrum and its phase spectrum. Let this complex spectral representation be called $S(\omega)$. Then

$$S(\omega) = \int_{-\infty}^{+\infty} s(t) e^{-j\omega t} dt = |S(\omega)| e^{+j\phi_s(\omega)}$$

where $|S(\omega)|$ represents the amplitude distribution and $\phi_s(\omega)$ represents the phase distribution. After the transmitted signal, s(t), strikes the reflecting object, a return signal, r(t), will be reflected back. This signal can also be expressed in the frequency domain. Thus,

$$R(\omega) = \int_{-\infty}^{+\infty} r(t)e^{-j\omega t}\,dt = |R(\omega)|e^{+j\phi_R(\omega)} .$$

Now $s(t) = m(t)\text{Cos}\ \omega_c t = m(t)\left[\frac{e^{+j\omega_c t} + e^{-j\omega_c t}}{2}\right]$, and

$$F[m(t)] = M(\omega) = \int_{-\infty}^{+\infty} m(t)e^{-j\omega t}\,dt ,$$

$$\text{also } F[s(t)] = S(\omega) = \int_{-\infty}^{+\infty} s(t)e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{+\infty} m(t)\left[\frac{e^{+j\omega_c t} + e^{-j\omega_c t}}{2}\right] e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{+\infty} \frac{m(t)}{2}\, e^{-j[\omega-\omega_c]t}\,dt + \int_{-\infty}^{+\infty} \frac{m(t)}{2}\, e^{-j[\omega+\omega_c]t}\,dt$$

$$= \frac{M(\omega-\omega_c)}{2} + \frac{M(\omega+\omega_c)}{2} \text{ . So,}$$

$$S(\omega) = \frac{1}{2}\left[M(\omega-\omega_c) + M(\omega+\omega_c)\right] .$$

Although $M(\omega)$ is a function which is exactly defined for <u>all</u> values of ω from $-\infty$ to $+\infty$, it can be replaced in a practical (band-limited) modulating system by a truncated function $\hat{M}(\omega)$ which is defined as:

$$\hat{M}(\omega) = M(\omega) \qquad |\omega| < W$$

$$\hat{M}(\omega) = 0 \qquad |\omega| \geq W.$$

In this definition the modulating system bandpass is assumed to be confined to the region $|\omega| < W$. A typical $|\hat{M}(\omega)|$ distribution is shown in Figure 1.

If one considers the transmitted signal to be $\hat{m}(t)\text{Cos}\ \omega_c t$ where $\hat{m}(t)$ is the inverse Fourier transform of $\hat{M}(\omega)$, the Fourier transform, $S(\omega)$, of the transmitted signal becomes

$$S(\omega) = \frac{1}{2}\,[\hat{M}(\omega-\omega_c) + \hat{M}(\omega+\omega_c)] \ .$$

A plot of $|S(\omega)|$ is shown in Figure 2. Note that both $|\hat{M}(\omega)|$ and $|S(\omega)|$ are even functions. This is a characteristic of Fourier transforms of real variables as shown in Appendix B.

The illumination of the object by the transmitted signal can be thought of as a composite of individual sinusoidal illuminations, i.e., each of the spectral components of $S(\omega)$ can be thought of as illuminating the object separately. Each of these spectral components is then given its own (but not necessarily unique) particular phase shift and attenuation. As a result of this, the spectrum of the return signal differs from that of the transmitted signal. Although the distribution of $|S(\omega)|$ is symmetrical in the vicinity of $+\omega_c$ and $-\omega_c$, the distribution of $|R(\omega)|$ will not have these symmetrical properties in general. In those special cases where the distributions of $|R(\omega)|$ are symmetrical in the vicinity of $\pm\omega_c$, the return signal, $r(t)$, [which is the inverse Fourier transform of $R(\omega)$] will be expressible as a simple amplitude-modulated carrier, i.e.,

$$r(t) = a_{\omega_c}(t)\text{Cos}(\omega_c t + \phi_{\omega_c}),$$

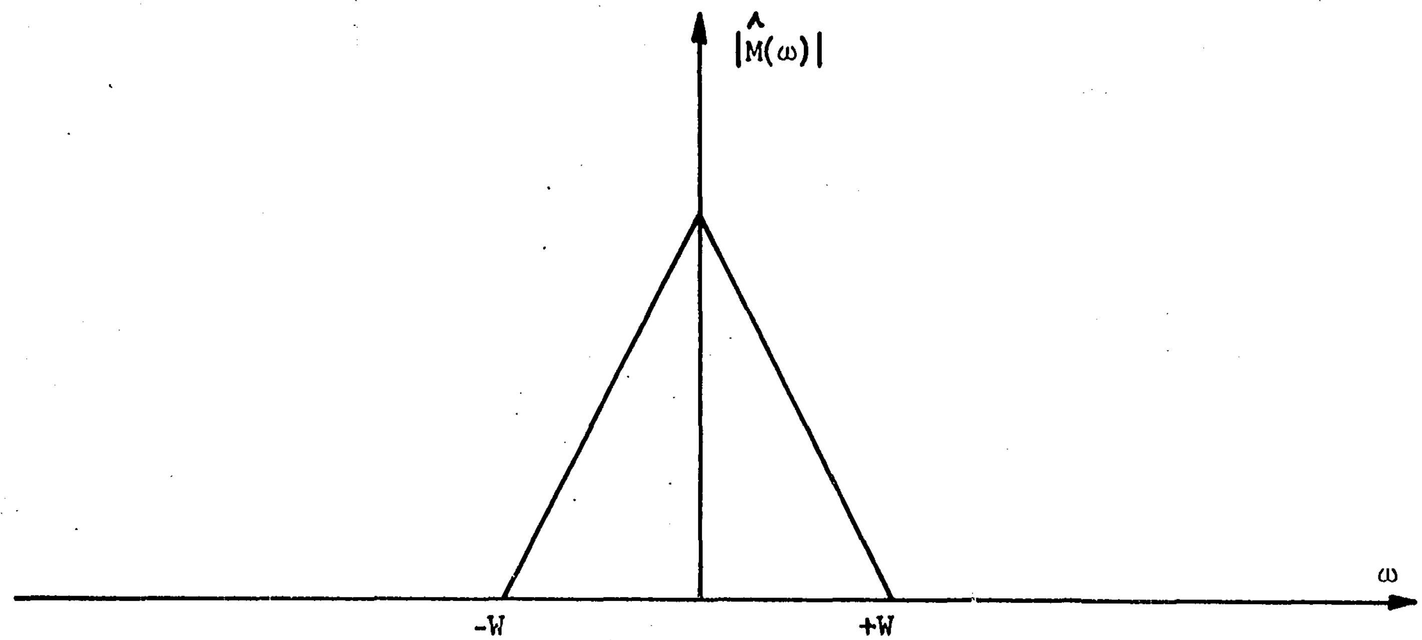


Figure 1. A typical $|\hat{M}(\omega)|$ distribution

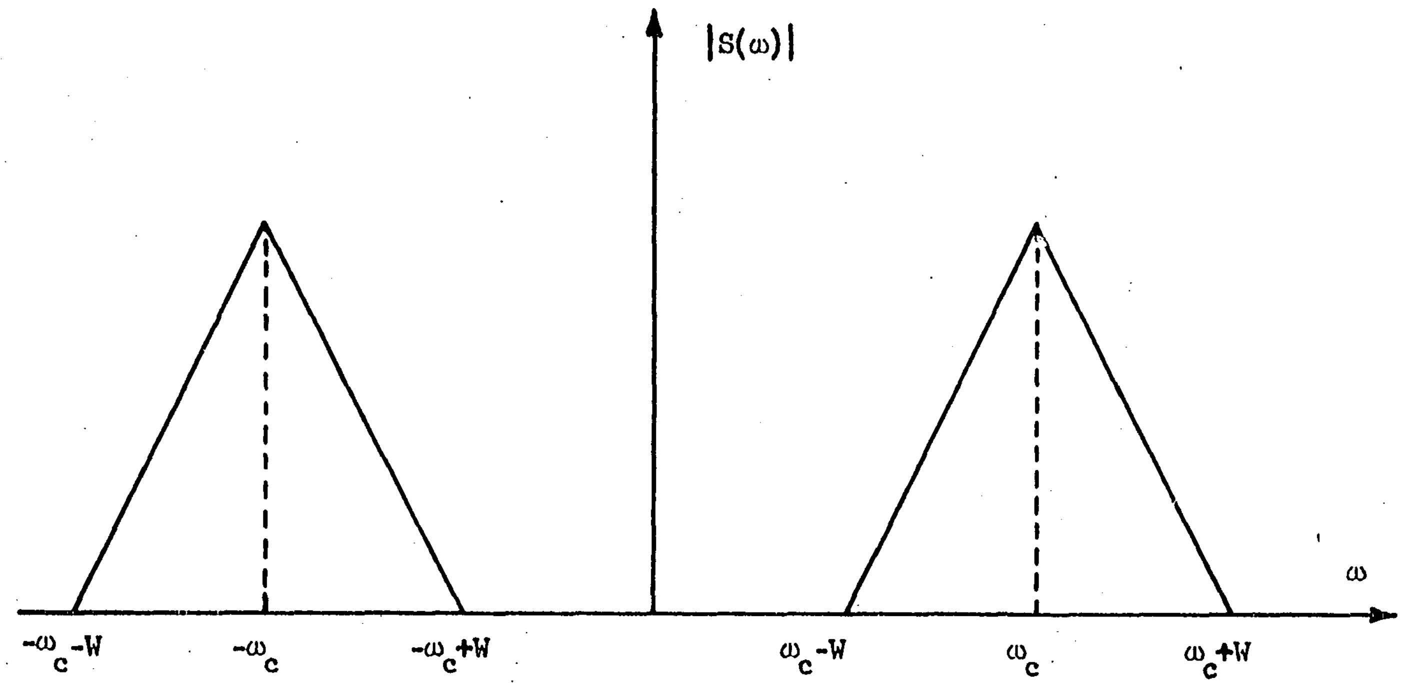


Figure 2. A typical $|S(\omega)|$ distribution

where $a_{\omega_c}(t)$ does not equal $m(t)$ in general, and ϕ_{ω_c} is the phase shift of the spectral line carrier, ω_c.

The general case, however, is illustrated in Figure 3; note that the distribution of $|R(\omega)|$ is not symmetrical in the vicinity of $\pm\omega_c$. Note, also, the evenness of $|R(\omega)|$. Because of the dissymmetry around $\pm\omega_c$, $r(t)$ cannot be expressed as a simple amplitude-modulated carrier. Instead its expression requires a phase-modulation term as well as an amplitude-modulation term (3, p. 168), i.e.,

$$r(t) = a_{\omega_c}(t)\text{Cos}\,[\omega_c t + \phi_{\omega_c}(t)]\ .$$

Note that in this expression the amplitude-modulating function and the phase-modulating function have ω_c subscripts. To understand the need for these subscripts, it is necessary to recall that for any particular ω_c, the transmitted-signal spectrum consists of the modulating spectrum distributed about $\pm\omega_c$ as shown in Figure 2. Increasing ω_c shifts the distributed spectrum away from the zero frequency axis and decreasing ω_c shifts the spectrum toward it. Thus the spectral components of a transmitted signal having a large ω_c value (although having the same relative distribution about the center frequency) lie considerably further away from the zero frequency axis than those of a transmitted signal having a low ω_c value. Thus, since the position of a spectral line on the frequency axis determines the amount of attenuation and phase shift that the line will undergo when reflected from a particular target, it is clear that the high group of transmitted frequencies will undergo a different operational process than the low group. This is illustrated in

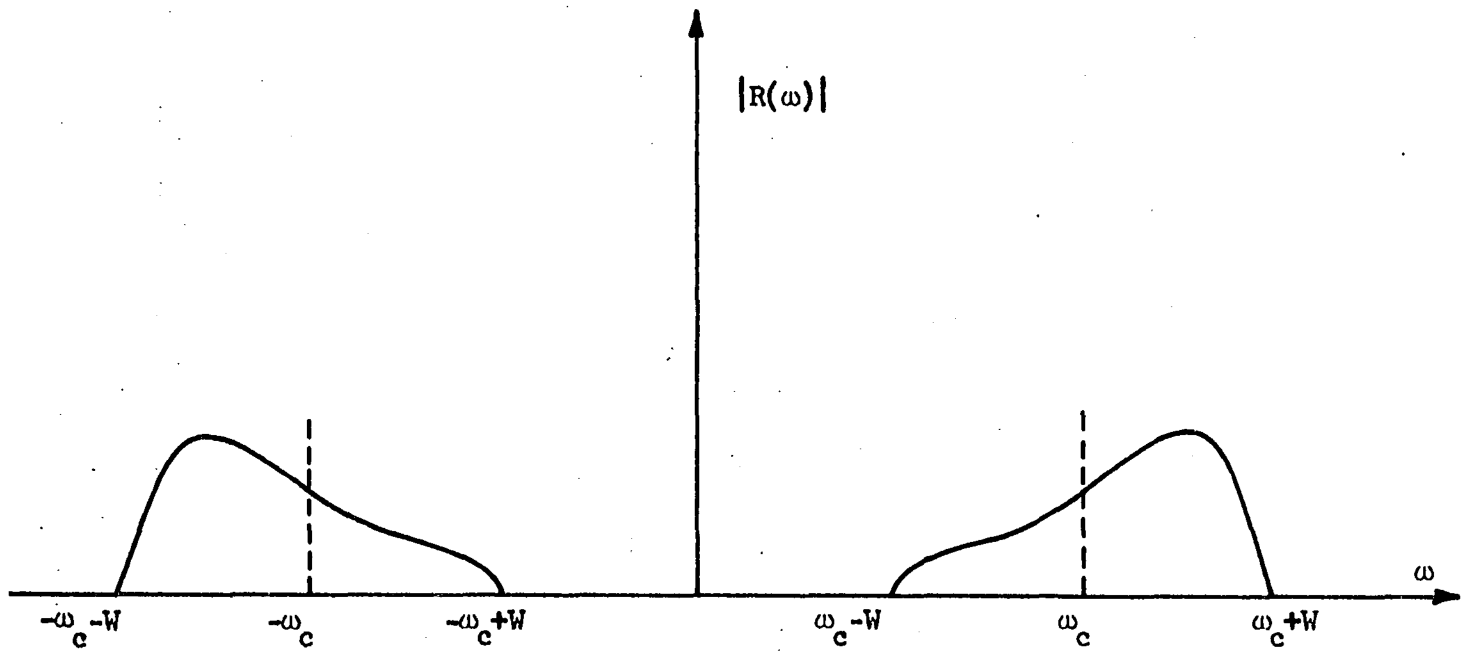
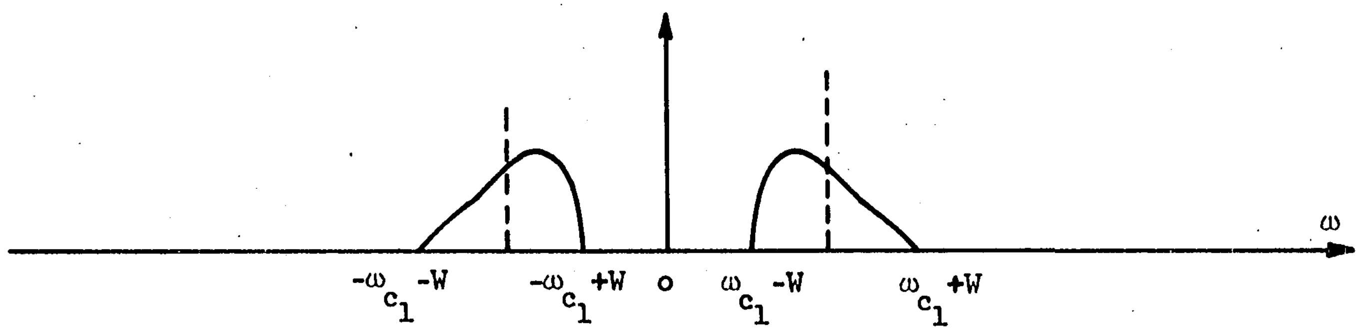


Figure 3. A typical $|R(\omega)|$ distribution

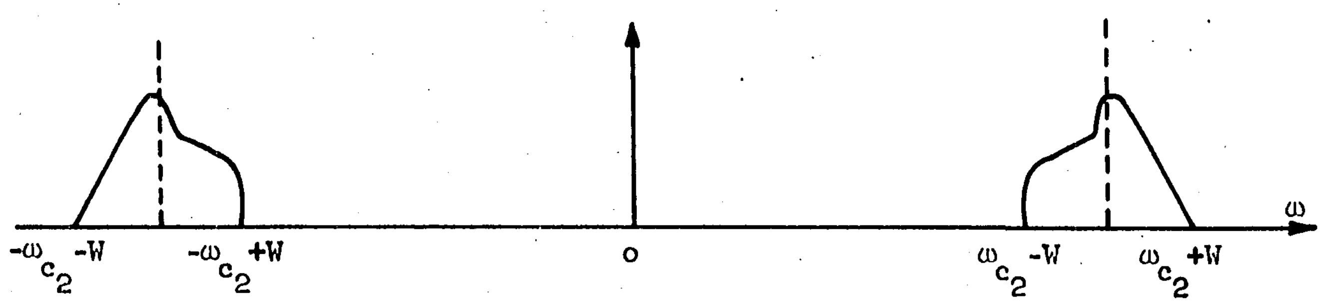
Figure 4.

Thus the value of the carrier frequency of a transmitted signal, in so far as it is responsible for the position of the spectral lines of the transmitted signal, is indirectly responsible for the operation that the lines undergo and for the resulting distribution, $R(\omega)$, in amplitude and phase of the returned signal components. Now the distribution of the returned spectral lines about the center frequency is a composite distribution which can be thought of as being composed of two component parts. One component can be associated with an amplitude-modulating function, $a(t)$, and the other can be associated with a phase-modulating function, $\phi(t)$. Since the total composite distribution is dependent upon the original carrier, the component distributions will also be dependent upon the carrier. Since $a(t)$ and $\phi(t)$ each correspond to the component distributions, they, too, are dependent upon the original carrier. Therefore, the need for the ω_c subscripts on $a_{\omega_c}(t)$ and $\phi_{\omega_c}(t)$ is established. It should be noted in passing that in spite of the fact that an amplitude-modulated function and a phase-modulated function are needed to describe the return signal, the same frequency components which exist in the transmitted spectrum also exist in the received spectrum; frequencies are neither created nor destroyed by the stationary target reflection process.

Because of this relatively simple difference between the frequency domain characteristics of the transmitted and reflected signals, and because the reflected signal has the same form as one emerging from an electrical network, namely



a. Distribution of $|R(\omega)|$ with low carrier transmission



b. Distribution of $|R(\omega)|$ with high carrier transmission

Figure 4. Effects of carrier frequency on $|R(\omega)|$ distributions

$$r(t) = a_{\omega_c}(t)\text{Cos}\,[\omega_c t + \phi_{\omega_c}(t)]\ ,$$

it seems quite reasonable to think of the stationary target as possessing a complex reflection-function, $F_t(\omega)$, similar in character to a network transfer-function.

Although useful insight is gained by noting that both reflecting objects and linear passive electrical networks give rise to return (or output) signals which have the form

$$r(t) = a_{\omega_c}(t)\text{Cos}\,[\omega_c t + \phi_{\omega_c}(t)]\ ,$$

the real argument for attributing a complex reflection-function, $F_t(\omega)$, to a reflecting object rests entirely upon the fact that the spectral character of the reflected signal differs in a rather special way from the spectral character of the transmitted signal. To examine this spectral modification fully, it is only necessary to consider that the illumination spectrum, $S(\omega)$, is entirely flat (i.e., constant) throughout the ω-domain and that $S(\omega)$ has a zero phase-angle at each point in the ω-domain. Stating this mathematically

$$S(\omega) \equiv 1$$

Now when each of the spectral components of $S(\omega)$ strikes the reflecting object it will undergo a certain (but not necessarily unique) amount of attenuation and phase shifting in the reflection process. The reflected signal, $R(\omega)$, [for $S(\omega) \equiv 1$] is therefore an exact measure of the spectral-modification property of the reflecting object. This measure will be defined as the complex reflection-function of the object, i.e.,

$$F_t(\omega) \triangleq R(\omega)\Big|_{S(\omega) = 1}$$

By taking the inverse Fourier transform, the impulsive response, y(t), of the reflecting object is obtained. Thus

$$y(t) \triangleq F^{-1}\left\{F_t(\omega)\right\} = F^{-1}\left\{R(\omega)\Big|_{S(\omega) = 1}\right\}$$

or,

$$y(t) \triangleq r(t)\Big|_{s(t) = \delta(t)}$$

where $\delta(t)$ is a unit impulse centered at $t = o$. The term special way is used in the above argument to mean that the target's reflecting character has a linear behavior, i.e., that the principal of superposition applies to the attenuation property at every point in the ω-domain and that the phase shifting property is not dependent upon the "strength" (i.e., the amplitude) of the illumination signal.

By defining $F_t(\omega)$ and y(t) successively in the manner outlined above, it is possible to go one step further and define an equivalent network transfer-function, $G_t(s)$, which can be used to represent the reflecting object. Such a transfer function will be defined here in the following way:

$$G_t(s) \triangleq L[y(t)]$$

The utility of this transfer function will be explained later. Let it suffice to say at this point that it will be found worthwhile to think of the stationary target in terms of an equivalent network which operates on the spectral components of any given $s(t)$ in such a way as to yield the spectral components of $r(t)$. Of course the impulsive response of the network and its Fourier transform are identical, respectively, to the impulsive response of the reflecting object and its Fourier transform. For this reason the same symbols, namely $y(t)$ and $F_t(\omega)$, will be used interchangeably to represent both the reflection process of the reflecting object and the transfer process of the equivalent network. Similarly the s-domain function, $G_t(s)$, will be used to characterise both the reflecting object and its network equivalent. To carry this one step further, the terms reflection function and transfer function will be used interchangeably in the discussion which follows because of the equivalence which has been established above.

The real value of describing a target in terms of a reflection function is obvious in terms of the simple relation which exists between input and output in the frequency domain and in the s-plane, i.e.,

$$R(\omega) = S(\omega)F_t(\omega)$$

and

$$R(s) = S(s)G_t(s) = \frac{1}{2}\,[M(s-j\omega_c) + M(s+j\omega_c)]\;G_t(s)$$

This relationship shows that once $F_t(\omega)$ or $G_t(s)$ is determined for a particular target, either by analytical or experimental technique, it can be used with many different S(s) or S(ω) distributions to determine the corresponding R(s) or R(ω) distributions, respectively, without the need for further experiment or measurement. Without such an expression relating general inputs to corresponding outputs, an experiment would have to be performed for each specific input and target combination in order to obtain a corresponding output. At this point, one might naturally ask if it is possible to obtain the reflection function of an arbitrary object by purely analytical techniques. The answer to this question in general is yes since it can be obtained, at least in principle, by considering a plane electromagnetic wave to impinge upon an object of interest and by solving the corresponding boundary value problem. However, the analytic difficulties (5, p. 453) encountered have prevented such solutions except for objects of rather simple geometry. It is therefore necessary to investigate methods for the experimental determination of reflection functions. It should be mentioned, however, that the analytical solutions which have been obtained for specific and simple geometries are somewhat useful in general since these results can be used to guide future measurement techniques. It is hoped in turn that information collected by experiment will be helpful in aiding future analytical solutions. More will be said about this later. The object at this point is simply to present in a developmental manner useful experimental methods.

E. Measurement of Stationary Target Transfer Function

Inspection of the second equation on page 19 shows that R(s) would be equal to G(s) if $\frac{1}{2}\,[M(s-j\omega_c) + M(s+j\omega_c)]$ were equal to unity. There is at least one modulation function, $m(t)$, namely the unit impulse, $\delta(t)$, for which this condition is satisfied. As shown in Appendix A, the Laplace transform of the unit impulse is unity, so if $m(t) = \delta(t)$, $M(s) = 1 = M(s \pm j\omega_c)$. With this reasoning it is apparent that if the carrier were modulated by a unit impulse the return signal, $s(t)$, would be the inverse Laplace transform of the target transfer-function, $G_t(s)$. Such modulation would therefore provide a means for the direct measurement of the target transfer-function. Two immediate shortcomings are apparent however. First, if modulation with a unit impulse were actually possible, the carrier, $\cos(\omega_c t)$, would be sampled just once (at its maximum value), and the carrier, as such, would not be transmitted. Thus, the system would not be taking advantage of the r-f (radio-frequency) carrier principle. The second difficulty lies in the impossibility of generating a true unit impulse. The closest thing to a unit impulse is a pulse of extremely narrow width and extremely large amplitude. If such a pulse were used as a modulating function, advantage would be taken of the carrier principle. Thus, from an intuitive viewpoint a short-pulse system seems appropriate. To justify this notion analytically, it is necessary to determine the Laplace transform of a pulse type modulating function. This transform, obtained in Appendix A, is expressed as

$$P(s) = \left[\frac{1 - e^{-as}}{as}\right]$$ where " $\frac{1}{a}$ " is the impulse amplitude and "a" is the pulse width.

Since $R(s) = \frac{1}{2}\left[M(s-j\omega_c) + M(s+j\omega_c)\right] G_t(s)$, it is clear that, with $M(s) = P(s)$,

$$R(s) = \frac{1}{2a}\left[\frac{1-e^{-a(s-j\omega_c)}}{s-j\omega_c} + \frac{1-e^{-a(s+j\omega_c)}}{s+j\omega_c}\right] G_t(s)$$

$$= \frac{G_t(s)}{2a}\left\{\frac{\left[1-e^{-a(s-j\omega_c)}\right](s+j\omega_c) + \left[1-e^{-a(s+j\omega_c)}\right](s-j\omega_c)}{s^2+\omega_c^2}\right\}$$

$$= \frac{G_t(s)}{2a}\left\{\frac{2s - se^{-as}\left[e^{+j\omega_c a} + e^{-j\omega_c a}\right] - j\omega_c e^{-as}\left[e^{+j\omega_c a} - e^{-j\omega_c a}\right]}{s^2+\omega_c^2}\right\}$$

$$= \frac{G_t(s)}{2a}\left\{\frac{2s - 2e^{-as}\left[s\,\mathrm{Cos}(\omega_c a) - \omega_c\,\mathrm{Sin}(\omega_c a)\right]}{s^2+\omega_c^2}\right\}$$

$$= \frac{G_t(s)}{a}\left\{\frac{s - e^{-as}\left[s\,\mathrm{Cos}\,(\omega_c a) - \omega_c\,\mathrm{Sin}\,(\omega_c a)\right]}{s^2+\omega_c^2}\right\}$$

Now to investigate $R(s)$ for small a, the limit of $R(s)$ as $a \to o$ is taken:

$$\lim_{a\to o} R(s) = \lim_{a\to o}\left[\frac{1}{a}\left\{\frac{s-e^{-as}[s\,\mathrm{Cos}\,(\omega_c a) - \omega_c\mathrm{Sin}(\omega_c a)]}{s^2+\omega_c^2}\right\}\right] G_t(s)$$

Although this takes the indeterminate form o/o, application of L'Hospital's Rule gives:

$$\lim_{a\to o} R(s) = \lim_{a\to o}\left\{\frac{o + se^{-as}\left[s\,\mathrm{Cos}(\omega_c a) - \omega_c\,\mathrm{Sin}(\omega_c a)\right]}{s^2+\omega_c^2}\right.$$

$$\left. + \frac{-e^{-as}\left[-\omega_c s\,\mathrm{Sin}\,(\omega_c a) - \omega_c^2\,\mathrm{Cos}\,(\omega_c a)\right]}{s^2+\omega_c^2}\right\} G_t(s)$$

$$= \lim_{a \to 0} \left\{ \frac{s^2 + \omega_c^2}{s^2 + \omega_c^2} \right\} \left\{ G_t(s) \right\} = 1 \left\{ G_t(s) \right\} .$$

So,

$$\lim_{a \to 0} R(s) = G_t(s) .$$

Thus, in the limit, $R(s) = G_t(s)$, as one would intuitively expect. Unfortunately, it is difficult to estimate how close the functional form of $R(s)$ approaches the functional form of $G_t(s)$ by substitution of finite nonzero values for "a" and "ω_c" in the above equation. However, the limiting process shown above does aid in justifying the intuitive notion of using short-pulse modulation to obtain an approximate measure of $G(s)$.

Presuming that $R(s)$ and $G_t(s)$ can be made essentially equal by selection of a proper modulating function, the determination or measurement of $G_t(s)$ becomes a straight-forward radio detection and computation problem. To obtain $G_t(s)$, the Laplace transform of $r(t)$ must be taken. This can be done in at least two different ways. If apparatus is available for direct measurement or direct recording of $r(t)$, the computation of $R(s)$ can be performed directly. If response difficulties prevent direct measurement of $r(t)$, a combination AM and PM detection scheme can be used to obtain $a(t)$ and $\phi(t)$. These functions together with ω_c can be used to reconstruct the functional form of $r(t)$, and the transform can then be taken. The computation can be performed by either digital or analog computer techniques or by a combination of both. Also the computations can be performed either in real time or in machine time whichever is appropriate to the application.

Another way to obtain an approximate measure of the stationary-target transfer-function, $F_t(\omega)$, [recall that $G_t(j\omega) = F_t(\omega)$] is to sample the frequency spectrum of the target. To do this, a set of c-w radars can be used. As shown in Appendix C, each c-w radar operating at a unique frequency gives rise to a pair of discrete lines (impulses) in the frequency domain; one line is located at $+\omega_c$ and the other at $-\omega_c$. By using a set of these radars with carrier frequencies separated by a proper interval, sampling impulses are produced in the frequency domain which are separated by a proper sampling interval. To determine this proper sampling interval it is necessary to refer to the sampling theorem in the frequency domain. Consider, again the object's impulse-response function, y(t), which has a Fourier transform exactly equal to the frequency spectrum of the object being observed. Since the target's impulsive response exists during a finite period (say T seconds long) only, the function, y(t), need only be specified during this finite period. Now any function completely specified within a finite interval, T, can be represented exactly within that interval by a Fourier series of terms with fundamental frequency, 1/T. This series consists of an infinite number of harmonics each of which exists in the frequency domain as a pair of impulses. Also each harmonic frequency is separated from its neighbor by 1/T cps. Thus, the sampling theorem (4, p. 71) reasons that an infinite number of sampling impulses separated by 1/T cps are sufficient to completely specify the function, y(t). Applying this reasoning to the situation at hand, it becomes clear that the object's frequency spectrum can be completely specified by using an infinite number of sampling impulses spaced 1/T cycles apart or $2\pi/T$

radians apart. By using a finite set of samples, i.e., a finite set of c-w radars, only a finite portion of the frequency spectrum can be sampled. However, practical interest is always confined to a finite portion of the completely infinite spectrum so little generality is lost by this restriction. More will be said about this later.

F. Determination of Physical Realizability

After the transfer function, $G_t(s)$, [or $F_t(\omega)$] has been measured it may be desirable to perform a check on its physical realizability. One requirement for the physical realizability of $G_t(s)$ is that the impulse response, $y(t)$, of the system must be zero for negative time, i.e., $y(t) = L^{-1} [G_t(j\omega)] = 0$ for $t < 0$. Also $y(t)$ must approach zero as $t \rightarrow \infty$. If these two conditions on $y(t)$ are satisfied in the time domain, the physical realizability of $G_t(s)$ is assurred (4, p. 225).

Corresponding statements can be made in the s-plane and in the ω domain. Specifically, a necessary and sufficient criterion is the Paley-Wiener criterion (4, p. 226) which assures physical realizability if and only if the integral

$$I = \int_0^\infty \frac{\left| \text{Log } |G_t(j\omega)| \right| d\omega}{1 + \omega^2} = \int_0^\infty \frac{\left| \text{Log } |F_t(\omega)| \right| d\omega}{1 + \omega^2}$$

has a finite value.

The s-plane statement, lends itself as a mathematical type test for physical realizability, whereas the time domain statements are useful in a plausibility argument. Such a plausibility argument will now be stated to indicate that <u>all</u> reflecting objects <u>should</u> yield transfer functions which are physically realizable. The reasoning is as follows: If a unit impulse could actually be used as a modulating function, if antennas were capable of handling such wide-band information without distortion and if, further, the transmitting medium gave uniform attenuation without distortion, the return signal, $r(t)$, would be a true measure of the impulse response of the target transfer-function. Obviously if the impulse were not transmitted until some time, $t = o$, the target could not reflect before $t = o$. Thus the response, $y(t)$, could not begin before $t = o$. (This is true even if the target and radar are effectively separated by zero range by compensating for the round-trip transmission-time). Since $y(t)$ cannot occur before $t = o$, and since $y(t)$ is the impulse response of $G(s)$, one of the time domain criteria for physical realizability is satisfied. The other criterion, namely that $y(t) \rightarrow o$ as $t \rightarrow \infty$, is also felt to be satisfied since no known reflector has regenerative properties and all known reflectors have at least some small energy absorbing or dissipating characteristic. Thus $r(t)$ should $\rightarrow o$ as $t \rightarrow \infty$. Unless some fallacy can be found in the foregoing reasoning, it will be presumed that all stationary, passive targets give rise to physically realizable transfer functions. However, for any target which might cause skepticism, the mathematical conditions outlined at the beginning of this section can be applied to provide definite confirmation of physical realizability.

II. UTILITY

A. Introductory Comments

The preceding discussion has served a multiple purpose. First it served to introduce the notion of a complex reflection function which can be used to characterise the spectral behavior of a reflecting object. Secondly it showed the relationship between this spectral representation and the impulsive response of the reflecting object, namely that the two functions form a Fourier or Laplace transform pair. Thirdly it served to point out the correspondence between the object's reflection function and an equivalent network transfer-function which is physically realizable. Lastly it was able to indicate techniques by which the reflection function could be measured either indirectly by measuring the impulsive response or directly by sampling in the frequency domain.

The purpose of this section is to point out the utility of the reflection function and its corresponding equivalent network. In doing this, further comments can be made concerning the character of the reflection function and its measurement.

B. Object Identification by Correlation with Elementary Shapes

To begin with, it will be recalled from an earlier statement that the really basic utility of the reflection function, $F_t(\omega)$, [or $G_t(s)$] lies in its ability to be used with an arbitrary transformed illumination signal, $S_i(\omega)$, to find the corresponding transformed reflected signal, $R_i(\omega)$, i.e.,

$$R_i(\omega) = S_i(\omega)\, F_t(\omega) \qquad \text{in Fourier form}$$

or

$$R_i(s) = S_i(s)\ G_t(s) \qquad \text{in Laplace form.}$$

Now suppose, for example, that the reflection functions have been measured and catalogued for several objects of rather elementary geometric shape such as a sphere, a cylinder, a rod, a bar, a cube, etc. Suppose further that an equivalent electrical network is built up to represent each of the elementary shapes. Now suppose that an arbitrary illumination signal, $S_i(\omega)$, is used to illuminate an unknown reflecting target. Let the same illumination signal be passed through each of the several networks which correspond to elementary geometric shapes. Then let the return, $r_u(t)$, from the unknown target be correlated with each of the several outputs of the elementary shape networks. If the unknown target is predominately spherical a good correlation between its return, $r_u(t)$, and the output of the sphere-type network will result. Similar comments can be made regarding the other elementary shapes. In other words, by correlating the return from an unknown target with returns that would be obtained from targets of pre-described and known shapes, the geometrical character of unknown targets can be learned -- at least in an approximate sense. Thus the representation of reflecting objects by equivalent networks is seen to have at least one use-ful application. Admittedly, such a technique could conceivably involve an extremely elaborate array of elements with corresponding equivalent net-works; but, none the less, the possibility is apparent. As a matter of fact, when operating in a real-time system with target recognition and iden-tification of paramount importance such a scheme could be most worth-while in spite of its complexity.

A block diagram of a typical system is shown in Figure 5. Note that the scheme shown there requires "n" cross correlators, each of which could conceivably contain a complex piece of computing equipment. Fortunately, however, the correlations (which are of prime importance in the recognition process) can be handled simultaneously during normal radar tracking operation without the need for complex computing equipment. Such correlation is carried out by matched-filter network methods.

C. Matched-Filter Correlation

To understand the process of network-type correlation-techniques (3, p. 232), consider an arbitrary catalogued return, $r_i(t)$, having Fourier transform, $R_i(\omega) = |R_i(\omega)|\, e^{+j\phi_{R_i}(\omega)}$. Consider also a filter with a transfer function, $G(\omega)$, defined by

$$G(\omega) = R_i^*(\omega)e^{-j\omega t_o} = |R_i(\omega)|\, e^{-j\,[\phi_{R_i}(\omega) + \omega t_o]} ,$$

where $R_i^*(\omega)$ is the complex conjugate of $R_i(\omega)$ and $e^{-j\omega t_o}$ is a phase shift factor to be discussed later. If the return signal, $R_i(\omega)$, is passed through the filter, the output, $o(\omega)$, can be expressed as

$$o(\omega) = R_i(\omega) \cdot R_i^*(\omega)e^{-j\omega t_o} .$$

Multiplying through by $e^{+j\omega t_o}$ gives

$$o(\omega)e^{+j\omega t_o} = R_i(\omega) \cdot R_i^*(\omega) .$$

The corresponding time domain representation is

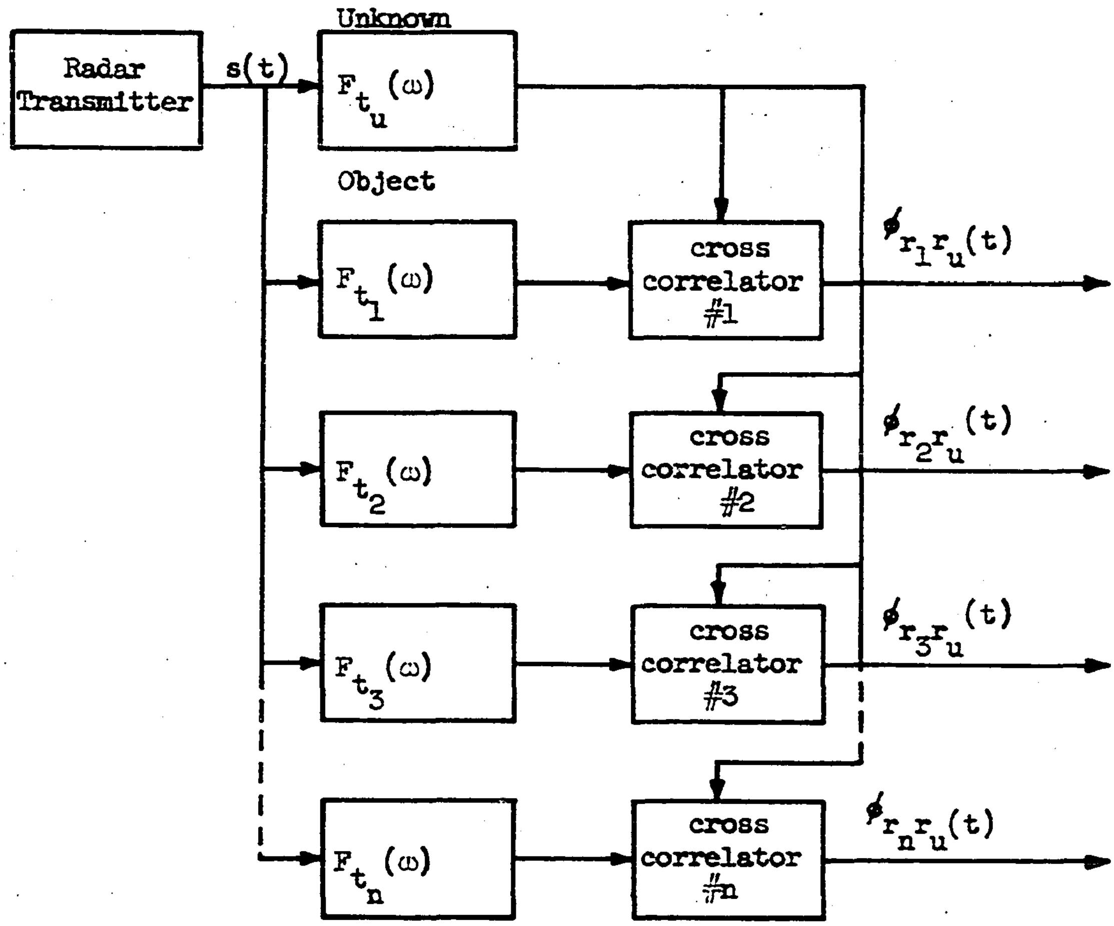


Figure 5. Block diagram of target recognition system

$$o(\tau + t_o) = \int_{-\infty}^{+\infty} r_i(x) r_i^*(\tau - x) dx ,$$

where $r_i(t)$ is the time domain representation of $R_i(\omega)$, and $r_i^*(t)$ is the time domain representation of $R_i^*(\omega)$.

It is shown in Appendix D that $r_i^*(t) = r_i(-t)$, so $r_i^*(\tau-x) = r_i(x-\tau)$. Therefore,

$$o(\tau + t_o) = \int_{-\infty}^{\infty} r_i(x) r_i(x-\tau) dx.$$

$$= \int_{-\infty}^{\infty} r_i(x-\tau) r_i(x) dx.$$

By letting $x-\tau = y$, it is clear that $x = y + \tau$, and $dx = dy$. Thus,

$$o(\tau + t_o) = \int_{-\infty}^{\infty} r_i(y) r_i(y + \tau)\, dy = \phi_{r_i r_i}(\tau).$$

The right hand side of this expression is seen to be the finite-autocorrelation function, $\phi_{r_i r_i}(\tau)$. By making another change in variable, namely $t = \tau + t_o$, it is clear that

$$o(t) = \int_{-\infty}^{\infty} r_i(y) r_i(y + t - t_o) dy = \phi_{r_i r_i}(t - t_o).$$

Now $\phi_{r_i r_i}(t)$ is the finite auto-correlation function and exists as an output only when the input signal is the signal to which the filter has been matched in its design. If the input signal is not the one to which the filter has been matched, the output will be the finite cross-correlation function. To see this mathematically, consider a filter matched to a particular return, $r_i(t)$, with Fourier transform, $R_i(\omega)$. With the filter matched to $r_i(t)$ its transfer function will be given by

$$G(\omega) = R_i*(\omega)e^{-j\omega t_o} .$$

Now consider that an unknown return signal, $R_u(\omega)$, enters the filter. The output will be

$$o(\omega) = R_u(\omega) \left[R_i*(\omega)e^{-j\omega t_o}\right] .$$

Thus,

$$o(\omega)e^{+j\omega t_o} = R_u(\omega) R_i*(\omega) .$$

Hence,

$$o(\tau + t_o) = \int_{\infty}^{+\infty} r_u(x) r_i*(\tau - x)dx.$$

However, $r_i*(t) = r_i(-t)$. Consequently,

$$o(\tau + t_o) = \int_{-\infty}^{+\infty} r_u(x) r_i(x-\tau)dx.$$

Now by letting $x - \tau = y$, we have

$$o(\tau + t_o) = \int_{-\infty}^{+\infty} r_u(y + \tau) r_i(y)dy.$$

This can be written as

$$o(\tau + t_o) = \int_{-\infty}^{+\infty} r_i(y)\, r_u(y + \tau)dy = \phi_{r_i r_u}(\tau).$$

Now if $\tau + t_o = t$, it is clear that

$$o(t) = \int_{-\infty}^{+\infty} r_i(y)\, r_u(y + t - t_o)dy = \phi_{r_i r_u}(t - t_o).$$

It is easily seen that in the special case where $u = i$, the finite cross-correlation function, $\phi_{r_i r_u}(t - t_o)$ becomes the finite auto-correlation function, $\phi_{r_i r_i}(t - t_o)$.

It is therefore seen that the output, $o(t)$, of the filter is either $\phi_{r_i r_i}(t - t_o)$ or $\phi_{r_i r_u}(t - t_o)$ depending upon whether or not the input is the one to which the filter is matched. These functions are just the finite-correlation functions (shifted to the right in time by an amount t_o) of the catalogued signal, $r_i(t)$. The effect of the phase shift factor, $e^{-j\omega t_o}$, used in the definition of $G(\omega)$, is now apparent. As a result of this phase shift factor, the output, $o(t)$, differs from $\phi_{r_i r_i}(t)$ by a simple translation in time. The choice of t_o must be made so as to make $G(\omega)$ a physically realizable network. This is the only restriction placed on the selection of a t_o value. Although earlier comments about physical realizability have been made, those comments had to do with the physical realizability of $F_t(\omega)$.

However,

$$G(\omega) = R^*(\omega)e^{-j\omega t_o} = [F_t(\omega) \cdot S(\omega)]^* e^{-j\omega t_o} = F_t^*(\omega)S^*(\omega)e^{-j\omega t_o}\,.$$

Thus

$$|G(\omega)| = |F_t^*(\omega)S^*(\omega)e^{-j\omega t_o}| = |F_t^*(\omega)|\,|S^*(\omega)|\,|e^{-j\omega t_o}| = |F_t(\omega)|\,|S(\omega)| \quad (1).$$

Now for physical realizability of $G(\omega)$ it is necessary and sufficient that the integral

$$I = \int_o^\infty \frac{\left|\text{Log } |G(\omega)|\right| d\omega}{1 + \omega^2}$$

have a finite value. However,

$$I = \int_o^\infty \frac{\left|\text{Log } |G(\omega)|\ d\omega\right|}{1 + \omega^2} = \int_o^\infty \frac{\left|\text{Log}|F_t(\omega)|\ |S(\omega)|\ |e^{j\omega t_o}|\right| d\omega}{1 + \omega^2}$$

$$I = \int_o^\infty \frac{\left|\text{Log } |F_t(\omega)| + \text{Log } |S(\omega)|\right| d\omega}{1 + \omega^2}$$

but $\left|\text{Log } |F_t(\omega)| + \text{Log } |S(\omega)|\right| \leq \left|\text{Log } |F_t(\omega)|\right| + \left|\text{Log } |S(\omega)|\right|$

so, $I \leq \displaystyle\int_o^\infty \frac{\left|\text{Log } |F_t(\omega)|\right| d\omega}{1 + \omega^2} + \int_o^\infty \frac{\left|\text{Log } |S(\omega)|\right| d\omega}{1 + \omega^2}$.

Now the first term on the right of this inequality has already been established to be finite since $F_t(\omega)$ corresponds to a physically realizable network. The second term must also be finite since $S(\omega)$ also corresponds to a physically realizable network. This is true since $s(t) = o$ for $t < o$ and $s(t) \to o$ as $t \to \infty$, since no illumination signal can be on forever. [Recall that $s(t) = F^{-1}\left\{S(\omega)\right\}$]. Thus the right-hand side of the inequality is finite. Moreover I is not negative since the numerator and denominator of its integrand are never negative. Thus,

$$o \leq I \leq \text{some finite number, } M$$

Therefore, I = a finite number, N, and so the physical realizability of $G(\omega)$ is assured. Thus if $g(t) = F^{-1}\{G(\omega)\}$, then $g(t) = o$ for $t \leq o$. Now,

$$g(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} g(\omega)e^{-j\omega t}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_i{}^*(\omega)e^{+j\omega(t-t_o)}\,d\omega = r_i{}^*(t-t_o)$$

$$= r_i(t_o-t).$$

Thus,

$$g(t) = r_i(t_o-t).$$

It is therefore apparent that for $g(t)$ to be zero for $t < o$, the return signal, $r_i(t)$, must be completely subsided by the time, t_o. In actuality, the signal return, $r_i(t)$, must first be examined to determine the period outside of which $r_i(t)$ essentially subsides. Knowing this, any time, t_o, which is greater than this period can be picked for use in the specification of the transfer function, $G(\omega)$. This assures that $r_i(t)$ subsides before $t = t_o$ and correspondingly assures that $G(\omega)$ will be physically realizable. Thus the system of Figure 5 can be replaced by the system of Figure 6. Note that the correlators are no longer needed. They have all been eliminated completely at the expense of one additional block namely the $S^*(\omega)e^{-j\omega t_o}$ block. Note that the $F_{t_i}(\omega)$ blocks have been replaced by $F_{t_i}{}^*(\omega)$ blocks. This adds no additional complexity since $|F(\omega)^*| = |F(\omega)|$

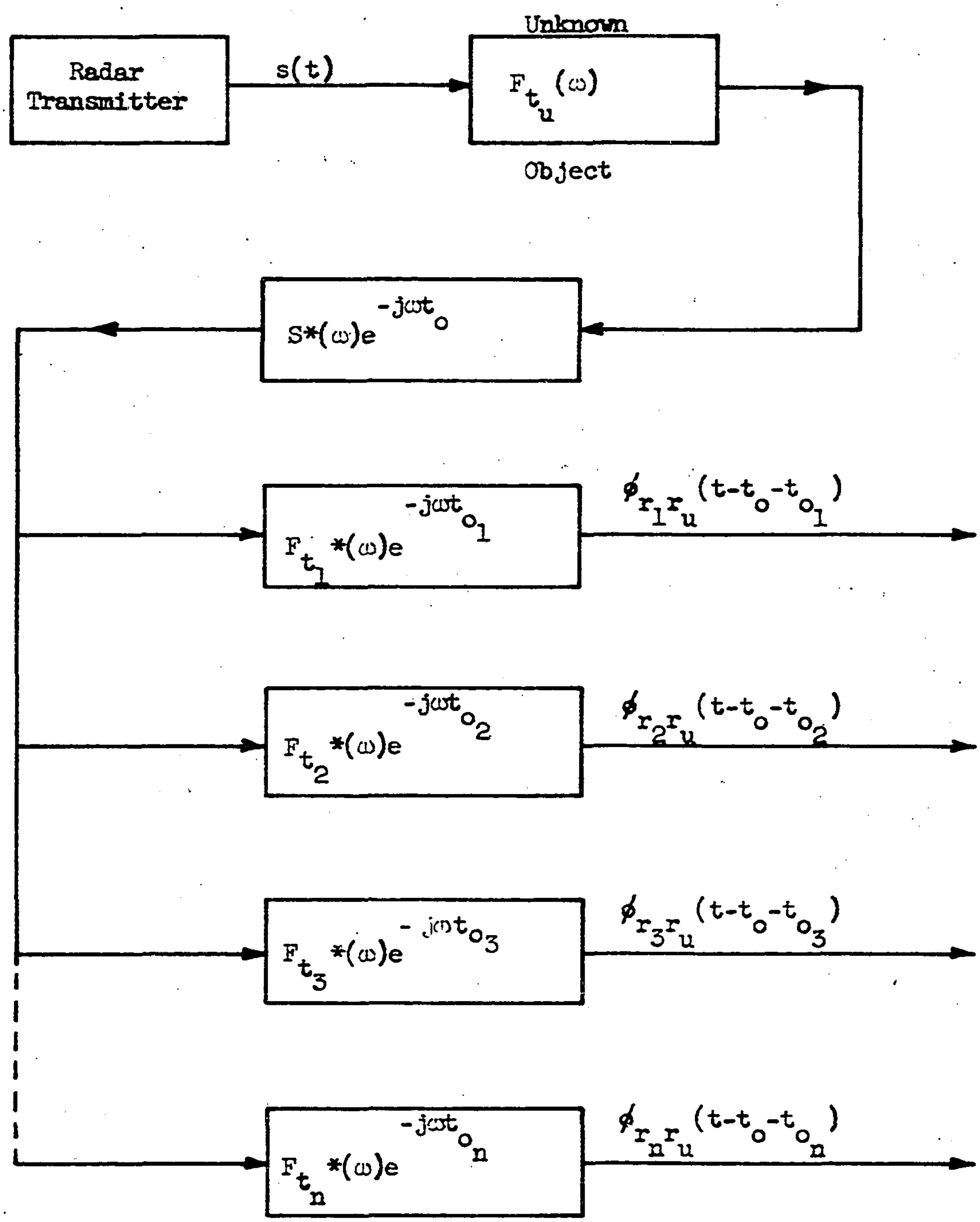


Figure 6. Block diagram of a target recognition system using matched filter correlators

and $\underline{/F^*(\omega)} = - \underline{/F}(\omega)$.

In order to provide a set of references with which to compare the various $\phi_{r_i r_u}(t - t_o)$, it is suggested that a set of $\phi_{r_i r_i}(t - t_o)$ be used. These are easily generated by using a set of $F_t(\omega)e^{-j\omega t_o}$ blocks. The use of such a set is illustrated in Figure 7.

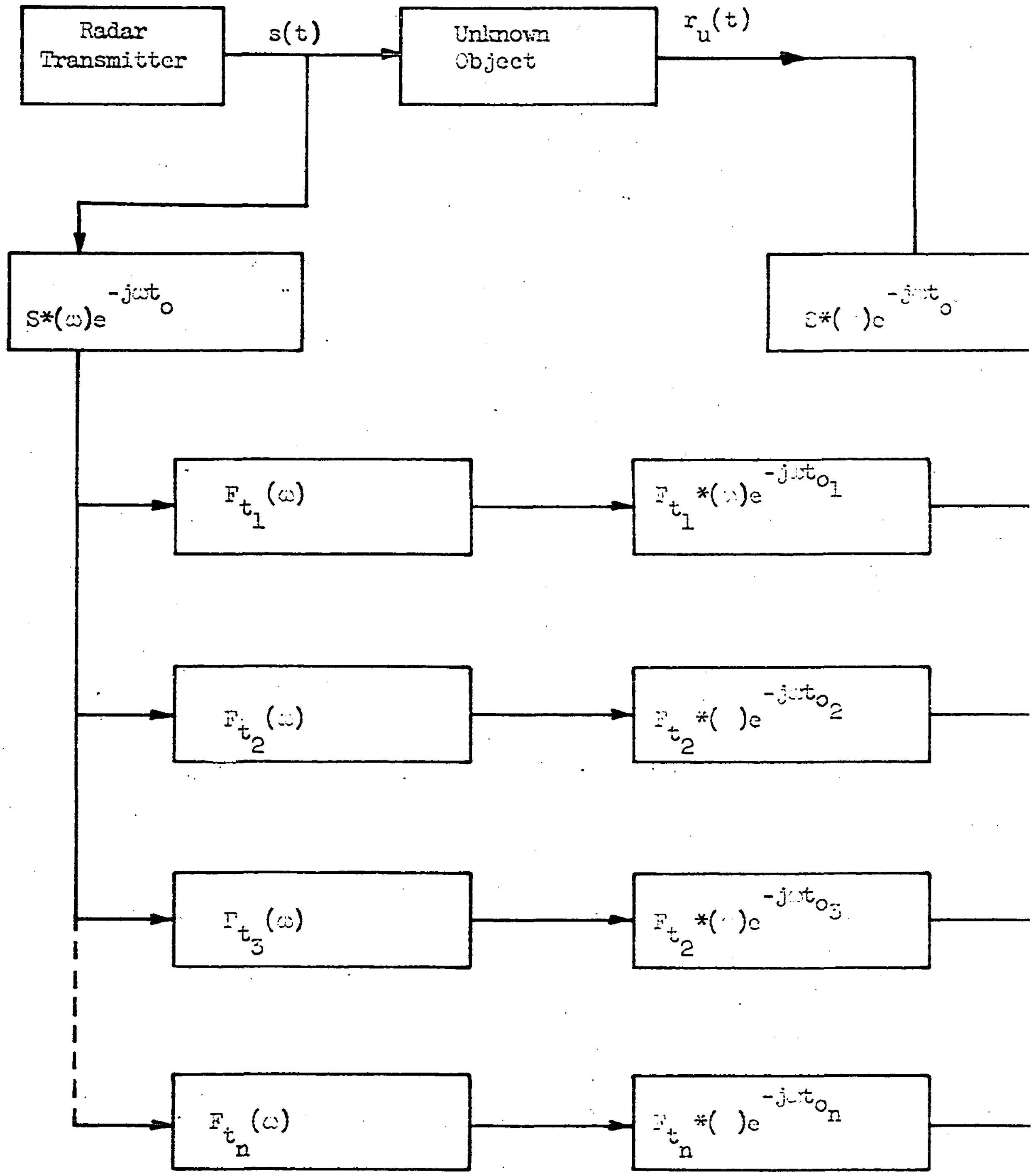
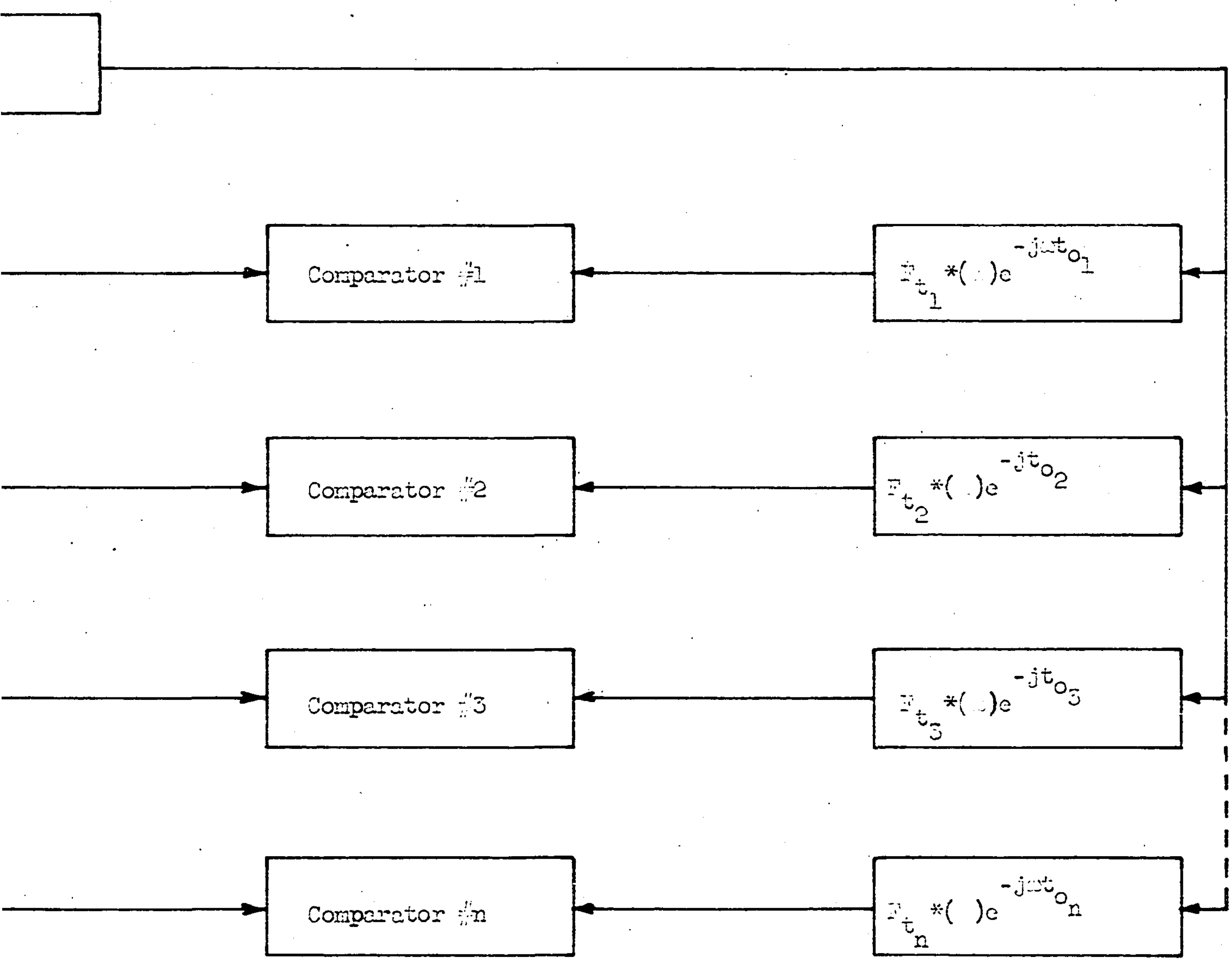


Figure 7. Partial block diagram of a target recognition system using comparators.



Comparator #1
Comparator #2
Comparator #3
Comparator #n
$F_{t_1}*(\ )e^{-j\omega t_{o_1}}$
$F_{t_2}*(\ )e^{-j\omega t_{o_2}}$
$F_{t_3}*(\ )e^{-j\omega t_{o_3}}$
$F_{t_n}*(\ )e^{-j\omega t_{o_n}}$

III. SYNTHESIS

It was pointed out in the introduction that any particular target's reflection function could be measured by at least two techniques. One involved the use of unit impulses or reasonable facsimiles thereof. The difficulties in attempting such measurements were pointed out and another method was suggested, the latter involving sampling in the frequency domain. Fortunately this second method not only allows exact measurements to be made relatively simply but also allows the data (which would be collected by this method) to be used immediately and simply in a synthesis scheme to realize a network equivalent. To understand this method completely and to see how it leads to a simple and direct synthesis procedure, consider any reflecting object and let its impulsive response, $y(t)$, be confined to the interval, $o \leq t \leq T$, as shown in Figure 8. Because $y(t)$ is confined to the interval $o \leq t \leq T$, it is possible to expand $y(t)$ in a Fourier series (3, p. 22) in that interval, i.e.,

$$y(t) = \sum_{n=-\infty}^{+\infty} C_n \, e^{\frac{j2n\pi t}{T}} \qquad o < t < T$$

$$= o \qquad o \geq t \geq T$$

or

$$y(t) = \sum_{n=-\infty}^{\infty} C_n \, e^{jn\omega_o t} \qquad o < t < T$$

$$= o \qquad o \geq t \geq T$$

where $$C_n = \frac{1}{T} \int_o^T y(t) \, e^{-jn\omega_o t}$$

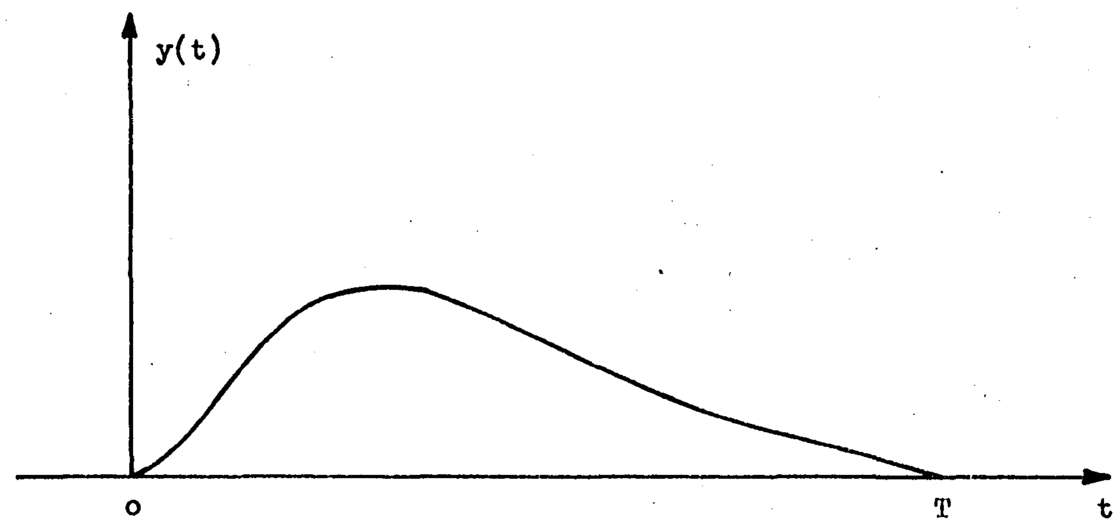


Figure 8. A typical impulsive response

and $\omega_o = \frac{2\pi}{T}$

but $F_t(\omega) = \int_{-\infty}^{+\infty} y(t)\, e^{-j\omega t}\, dt$

so $F_t(\omega) = \int_{o}^{T} y(t)\, e^{-j\omega t}\, dt$.

Therefore

$$F_t(n\omega_o) = \int_{o}^{T} y(t)\, e^{-j(n\omega_o)t}\, dt .$$

By comparing this expression for $F_t(n\omega_o)$ with the expression for C_n, it is seen that

$$C_n = \frac{1}{T} F_t(n\omega_o) .$$

Thus

$$y(t) = \sum_{n=-\infty}^{+\infty} \left[\frac{1}{T} F_t(n\omega_o) e^{jn\omega_o t}\right] \qquad o < t < T$$

$$= o . \qquad o \geq t \geq T$$

Now $F_t(\omega) = \int_{-\infty}^{+\infty} y(t)\, e^{-j\omega t}\, dt$

$$= \int_{o}^{T} \sum_{n=-\infty}^{+\infty} \frac{1}{T} F_t(n\omega_o) e^{jn\omega_o t}\, e^{-j\omega t}\, dt$$

$$= \sum_{n=-\infty}^{n=+\infty} \frac{1}{T} F_t(n\omega_o) \int_{o}^{T} e^{-j(\omega - n\omega_o)t}\, dt .$$

Now let $t = \hat{t} + T/2$ then $dt = d\hat{t}$, and when $t = o$, $\hat{t} = -T/2$. Also, when $t = T$, $\hat{t} = T/2$. It is now possible to write

$$F_t(\omega) = \sum_{n=-\infty}^{n=+\infty} \frac{1}{T} F_t(n\omega_o) \int_{-T/2}^{T/2} e^{-j(\omega-n\omega_o)(\hat{t}+T/2)} d\hat{t}$$

$$= \sum_{n=-\infty}^{+\infty} \frac{1}{T} F_t(n\omega_o) e^{-j(\omega-n\omega_o)T/2} \int_{-T/2}^{T/2} e^{-j(\omega-n\omega_o)\hat{t}} d\hat{t}$$

$$= \sum_{n=-\infty}^{n=+\infty} \frac{1}{T} F_t(n\omega_o) e^{-j(\omega-n\omega_o)T/2} \left[\frac{e^{-j(\omega-n\omega_o)}}{-j(\omega-n\omega_o)} \right]_{-T/2}^{T/2}$$

$$= \sum_{n=-\infty}^{+\infty} \frac{1}{T} F_t(n\omega_o) e^{-j(\omega-n\omega_o)T/2} \left[\frac{e^{+j(\omega-n\omega_o)T/2} - e^{-j(\omega-n\omega_o)T/2}}{j(\omega-n\omega_o)} \right]$$

$$= \sum_{n=-\infty}^{n=+\infty} \frac{1}{T} F_t(n\omega_o) e^{-j(\omega-n\omega_o)T/2} \frac{\left[\frac{e^{+j(\omega-n\omega_o)T/2} - e^{-j(\omega-n\omega_o)T/2}}{2j} \right]}{\frac{1}{T} \left[\frac{(\omega-n\omega_o)T}{2} \right]}$$

$$= \sum_{n=-\infty}^{+\infty} F_t(n\omega_o) e^{-j(\omega-n\omega_o)T/2} \left\{ \frac{\operatorname{Sin} [(\omega-n\omega_o)T/2]}{[(\omega-n\omega_o)T/2]} \right\} .$$

Therefore,

$$F_t(\omega) = \sum_{n=-\infty}^{+\infty} F_t(n\omega_o) e^{-j(\frac{\omega T}{2} - n\pi)} \left\{ \frac{\operatorname{Sin} [\frac{\omega T}{2} - n\pi]}{[\frac{\omega T}{2} - n\pi]} \right\} .$$

This expression shows that $F_t(\omega)$ is completely determined and expressible in terms of its values, $F_t(n\omega_o)$, at the sampling points. As a matter of fact it is a mathematical statement of the sampling theorem in the ω-domain and the mathematical development which proceeds that statement can be looked upon as a proof of the theorem.

Now to digress momentarily, consider the system shown in Figure 9. The transfer function, $G_t(s) = \frac{E_{out}(s)}{E_{in}(s)}$, is given by

$$G_t(s) = (1 - e^{-Ts})\left\{\frac{A(o)}{T}\frac{1}{s} + \sum_{n=1}^{\infty} \frac{2A(n\omega_o)}{T}\left[\frac{(-n\omega_o a_n) + sb_n}{s^2 + (n\omega_o)^2}\right]\right\}$$

where $\sqrt{a_n^{\,2} + b_n^{\,2}} = 1$ and $\tan^{-1}\frac{a_n}{b_n} = \phi(n\omega_o)$.

By taking the inverse Laplace transform, the impulsive response, y(t), is obtained:

$$y(t) = \frac{1}{T}[\mu(t) - \mu(t-T)]\left\{A(o) + 2\sum_{n=1}^{\infty} A(n\omega_o)\text{Cos}[n\omega_o t + \phi(n\omega_o)]\right\}$$

$$y(t) = \frac{1}{T}[\mu(t) - \mu(t-T)]\left\{A(o) + 2\sum_{n=1}^{\infty} A(n\omega_o)\left[\frac{e^{j[n\omega_o t+\phi(n\omega_o)]} + e^{-j[n\omega_o t+\phi(n\omega_o)]}}{2}\right]\right\}$$

$$y(t) = \frac{1}{T}[\mu(t) - \mu(t-T)]\left\{A(o) + \sum_{n=1}^{\infty} A(n\omega_o)\, e^{j[n\omega_o + \phi(n\omega_o)]} + \sum_{n=1}^{\infty} A(n\omega_o) e^{+j[-n\omega_o t - \phi(n\omega_o)]}\right\}$$

but $A(n\omega_o) = A(-n\omega_o)$ and $-\phi(n\omega_o) = \phi(-n\omega_o)$ so it is possible to write:

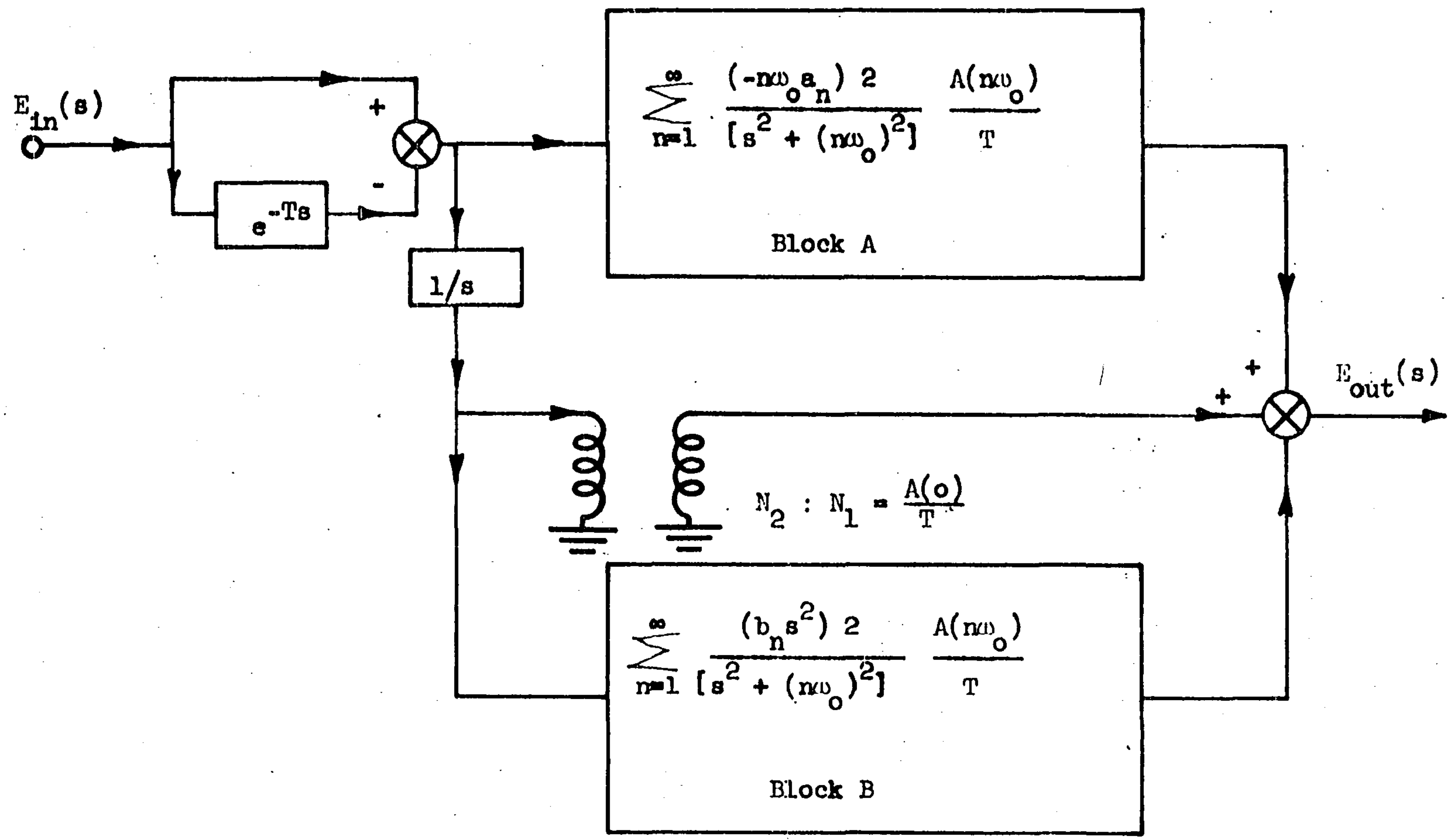


Figure 9. System block diagram

$$y(t) = \frac{1}{T}[\mu(t) - \mu(t-T)]\Big\{A(o) + \sum_{n=1}^{\infty} A(n\omega_o)e^{j[n\omega_o t + \phi(n\omega_o)]}$$

$$+ \sum_{n=1}^{\infty} A(-n\omega_o)e^{j[-n\omega_o t + \phi(-n\omega_o)]}\Big\}$$

$$= \frac{1}{T}[\mu(t) - \mu(t-T)]\Big\{A(o) + \sum_{n=1}^{\infty} A(n\omega_o)e^{j[n\omega_o t + \phi(n\omega_o)]}$$

$$+ \sum_{n=-1}^{-\infty} A(n\omega_o)e^{j[n\omega_o t + \phi(n\omega_o)]}\Big\}.$$

Now $\phi(o) = o$, so it is possible to write

$$A(o) = A(o)e^{j[o\omega_o + \phi(o\omega_o)]} .$$

Then it is also possible to write

$$y(t) \quad \frac{1}{T}[\mu\ (t) - \mu(t-T)]\Big\{\sum_{n=-\infty}^{+\infty} A(n\omega_o)e^{j[n\omega_o t + \phi(n\omega_o)]}\Big\}$$

or

$$y(t) = \frac{1}{T}\ [\mu(t) - \mu(t-T)]\Big\{\sum_{n=-\infty}^{\infty}\Big[A(n\omega_o)e^{+j\phi(n\omega_o)}\Big]\ e^{+jn\omega_o t}\Big\}\ .$$

However, $A(n\omega_o)e^{j\phi(n\omega_o t)} = F_t(n\omega_o)$, so

$$y(t) = \frac{1}{T}\ [\mu(t) - \mu\ (t-T)] \sum_{n=-\infty}^{+\infty} F_t(n\omega_o)\ e^{+jn\omega_o t} .$$

Taking the Fourier transform of $y(t)$ gives

$$F_t(\omega) = F\{y(t)\} = \frac{1}{T}\int_{-\infty}^{+\infty} [\mu\ (t) - \ \mu(t-T)] \sum_{n=-\infty}^{+\infty} F_t(n\omega_o)e^{+jn\omega_o t}\ e^{-j\omega t}\ dt$$

$$= \frac{1}{T} \sum_{n=-\infty}^{+\infty} F_t(n\omega_o) \int_{-\infty}^{+\infty} [\mu(t) + \mu(t-T)]\, e^{-j(\omega-n\omega_o)t}\, dt$$

$$= \frac{1}{T} \sum_{n=-\infty}^{+\infty} F_t(n\omega_o) \left[\int_0^T e^{-j(\omega-n\omega_o)t}\, dt \right].$$

By letting $t = \hat{t} + \frac{T}{2}$ the above expression can be written as

$$F_t(\omega) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} F_t(n\omega_o) \left[\int_{-T/2}^{T/2} e^{-j(\omega-n\omega_o)(\hat{t}-T/2)}\, d\hat{t} \right]$$

and after carrying out the integration,

$$F_t(\omega) = \sum_{n=-\infty}^{+\infty} F_t(n\omega_o) e^{-j[\frac{\omega T}{2} - n\pi]} \left\{ \frac{\mathrm{Sin}\, [\frac{\omega T}{2} - n\pi]}{[\frac{\omega T}{2} - n\pi]} \right\}.$$

This is seen to be the exact expression that was obtained for $F_t(\omega)$ in the development of the sampling theorem. Thus, if it is possible to synthesize the system of Figure 9, that system will serve as an exact network equivalent of the reflecting object considered at the outset of this discussion.

To see that this synthesis possibility does exist, at least in theory, it is only necessary to examine Figure 9 carefully on a block by block basis. To begin with, the blocks having transfer functions, e^{-Ts}, and, $\frac{1}{s}$, are immediately recognizable as a pure delay network and a pure integrating network respectively. Both can be synthesized (6, p. 18 and 8, p. 116) to any desired degree of accuracy by standard network synthesis techniques. The remaining blocks, namely blocks A and B, will now be considered.

Beginning with block A, it is immediately noted that since an infinite sum is needed to express this transfer function, it can be obtained by using an infinite number of sub-blocks each having a transfer function of the form

$$\frac{(-n\omega_o a_n)\ 2A(n\omega_o)}{[s^2 + (n\omega_o)^2]\ T}$$

, i.e., the first sub-block has transfer function:

$$G_{t_{A_1}} = \frac{[-(1)\omega_o a_1]\ 2A[(1)\omega_o]}{\{s^2 + [(1)\omega_o]^2\}\ T},$$

the second block has transfer function

$$G_{t_{A_2}} = \frac{[-(2)\omega_o]\ 2A[(2)\omega_o]}{\{s^2 + [(2)\omega_o]^2\}\ T},$$

and so on. By combining these blocks in the manner indicated in Figure 10, the overall transfer function of block A is obtained. Since all of the blocks have essentially the same functional form, they can each be synthesized by essentially the same kind of netowrk. For example, the n^{th} network is shown in Figure 11.

Block B is synthesized by using an infinite number of sub-blocks also, and these sub-blocks are combined in the same manner as the sub-blocks of block A. Again, since the transfer functions of the sub-blocks all have essentially the same form they can all be synthesized by essentially the same kind of network. For example, the n^{th} network of block B is shown in

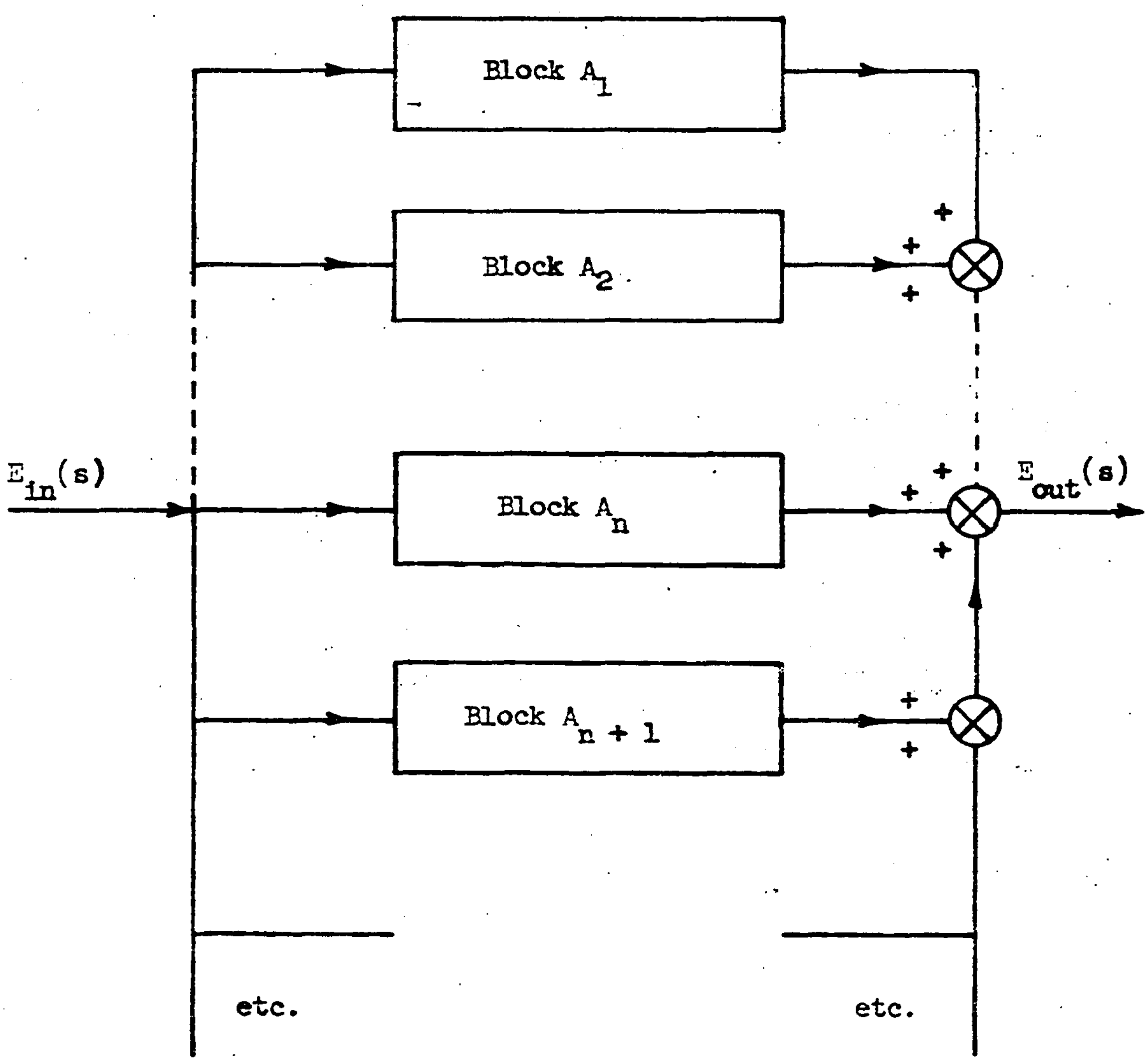
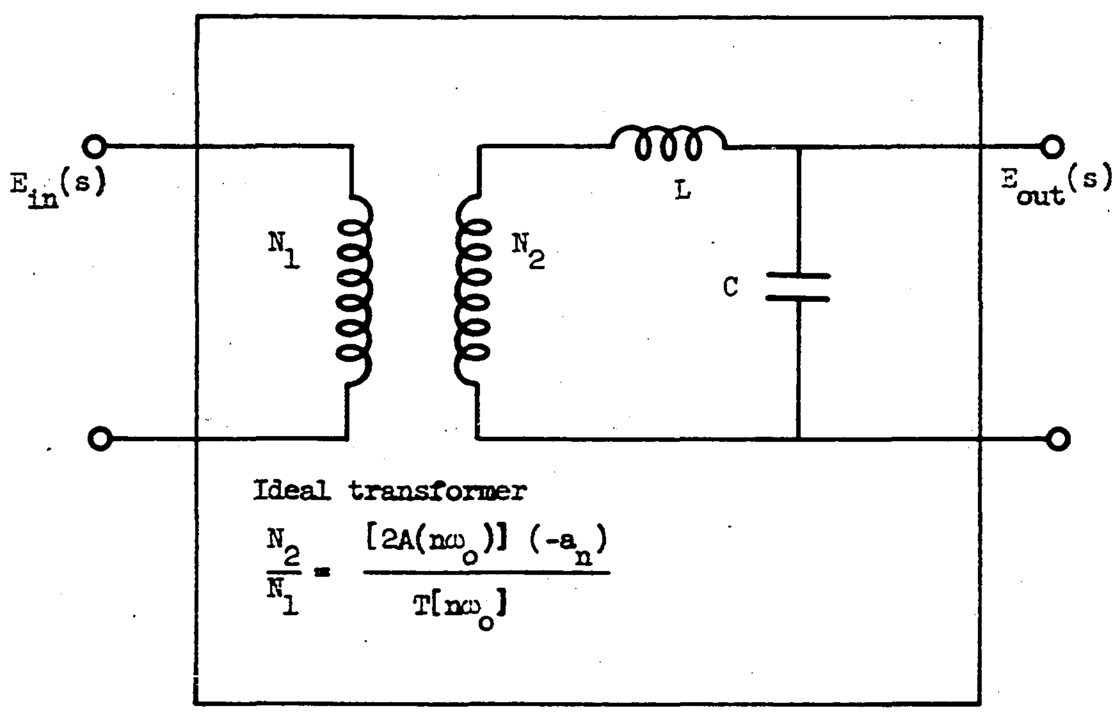


Figure 10. Block A as composed of its separate sub-blocks

$$LC = \frac{1}{(n\omega_o)^2}$$



$$G_{t_{A_n}}(s) = \frac{[2A(n\omega_o)][-n\omega_o a_n]}{T[s^2 + (n\omega_o)^2]}$$

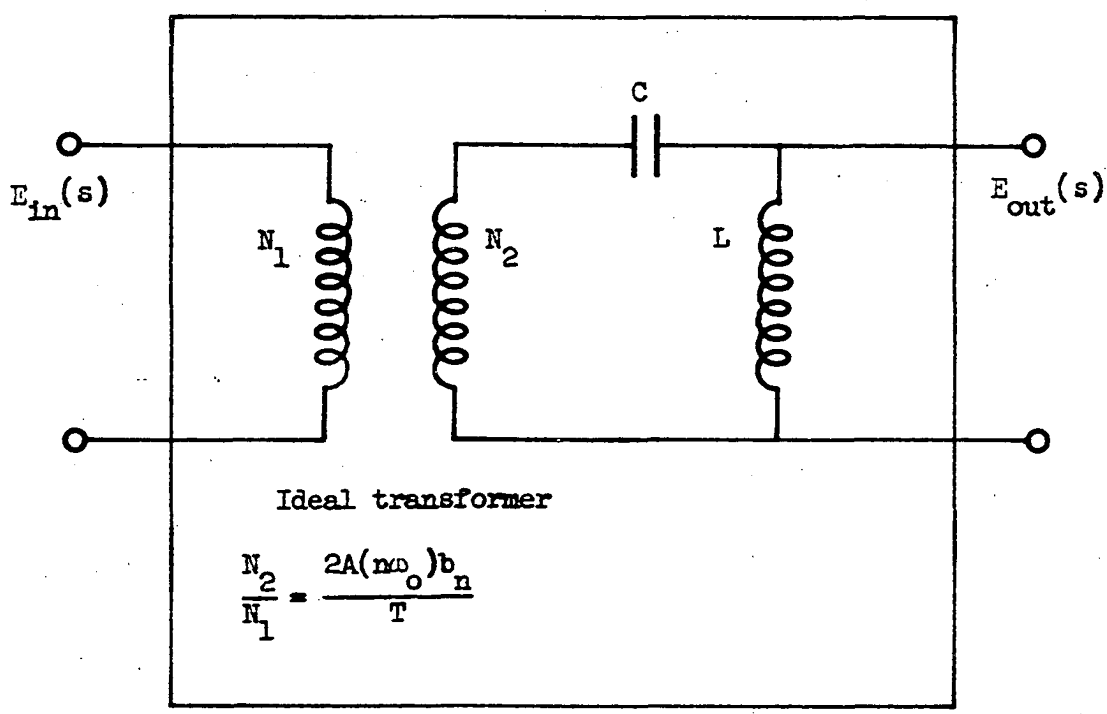
Figure 11. Block A_n as composed of electrical elements

Figure 12.

It is noted that a doubly infinite number of sub-blocks are required to provide an exact synthesis of the object's spectral character over the completely infinite spectrum, $o \leq \omega \leq \infty$. However, in practice, as was pointed out earlier, interest is usually confined to only a finite portion (say, $W_1 \leq \omega \leq W_2$) of the completely infinite spectrum. Also, one is usually content, when synthesizing arbitrary spectra, to provide an approximate fit rather than an exact fit. For example, if the synthesized spectrum is made to match the measured spectrum exactly at a sufficiently large number of points, a deviation of the synthesized spectrum from the desired spectrum is permissible between these points. It is this kind of synthesis that can be made in a relatively easy and straight-forward manner by using only a finite number of sub-blocks.

The synthesized spectrum can be made to match the object's measured spectrum exactly at the so-called sample points, and since these sample points are spaced sufficiently closely to preserve the informative character of the spectrum it is felt that such a synthesis procedure is both adequate and appropriate. Of course the effect of this procedure back in the time domain is to provide an approximate synthesis of the object's impulsive response, $y(t)$, over the interval, $o \leq t \leq T$, by using only a finite number of sine waves rather than an infinite number. This, of course, is quite typical of any practical synthesis procedure using Fourier methods. It is particularly suitable to the problem at hand because the interest here does not go beyond some upper frequency limit. For example, the radio frequency portion of the spectrum does not go beyond $\underline{6 \times 10^{10}\ \text{rad/sec}}$

$$LC = \frac{1}{(n\omega_o)^2}$$



$$G_{t_{B_n}}(s) = \frac{[2A(n\omega_o)][b_n]s^2}{[s^2 + (n\omega_o)^2]}$$

Figure 12. Block B_n as composed of electrical elements

(7, p. 32) and the radar frequency portion of the completely infinite spectrum does not go beyond 2×10^{11} rad/sec (7, p. 32). Although, interest may in fact begin at some $W_1 > o$ (for example $W_1 = 6 \times 10^4$ rad/sec and 6×10^8 rad/sec (7, p. 32) for the radio and radar portions of the spectrum, respectively), it may go down as low as zero radians per second, i.e., down to d-c. In those cases where $W_1 > o$, the Fourier sum will contain no frequencies below W_1, i.e., the d-c component, and those harmonics below W_1 will not be present. In any case where only a finite number of terms are taken, the synthesized y(t) is of course not equal to the actual impulsive response of the object under investigation; but since the interesting portion of the object's spectrum is confined to the region $W_1 \leq \omega \leq W_2$, the synthesized y(t) is just as useful in terms of the normal convolution procedures for obtaining outputs in the time domain, providing that the inputs that are to be convolved with the synthesized y(t) contain frequency terms that are also confined to the region $W_1 \leq \omega \leq W_2$. This is in fact the sort of consideration that makes it possible to specify a region of interest.

Typically the spectra of radio and radar transmission signals are confined to regions of this type. On the other hand, it is presumed that studies of the spectra of specific objects will illustrate those portions of the object's spectra that are most interesting, and the selection of appropriate operating regions can be made on this basis. Then, proper modulation signals can be selected so as to confine future transmitted spectra to those regions. This later approach is in fact another important justification for examining the spectra of specific objects. For example, from a radar tracking point of view it may be possible to optimize (at least in some sense such as minimizing bandwidths) the system operation by appropriately combining or matching modulation spectra to object spectra.

Having digressed momentarily to justify the use of finite Fourier sums, it is now possible to return to the problem at hand, namely that of using the finite number of sampled data in the synthesis scheme. To do this, it is convenient to proceed in the following way: Pick any one of the n sample-points, say the k^{th}. By using the amplitude, $A(k\omega_o)$, and the phase $\phi(k\omega_o)$ that exist at that sample point and by letting $a_k/b_k = \phi(k\omega_o)$ and $a_k^2 + b_k^2 = 1$, it is possible to solve immediately for a_k and b_k. Knowing these two constants, the value, $A(k\omega_o)$, and that the sampling interval, T, is related to ω_o by the equation $T = 2\pi/\omega_o$, the turns ratio, N_2/N_1, of Figures 11 and 12 can be computed. Also, by equating the LC product of the inductance and capacitance of those figures to $[1/k\omega_o]^2$, the blocks A_k and B_k are easily synthesized. This same procedure is then repeated at each sample point and the sub-blocks are thereby synthesized. The total synthesis is then completed by combining the sub-blocks with the delay line and integrator as illustrated in Figure 13.

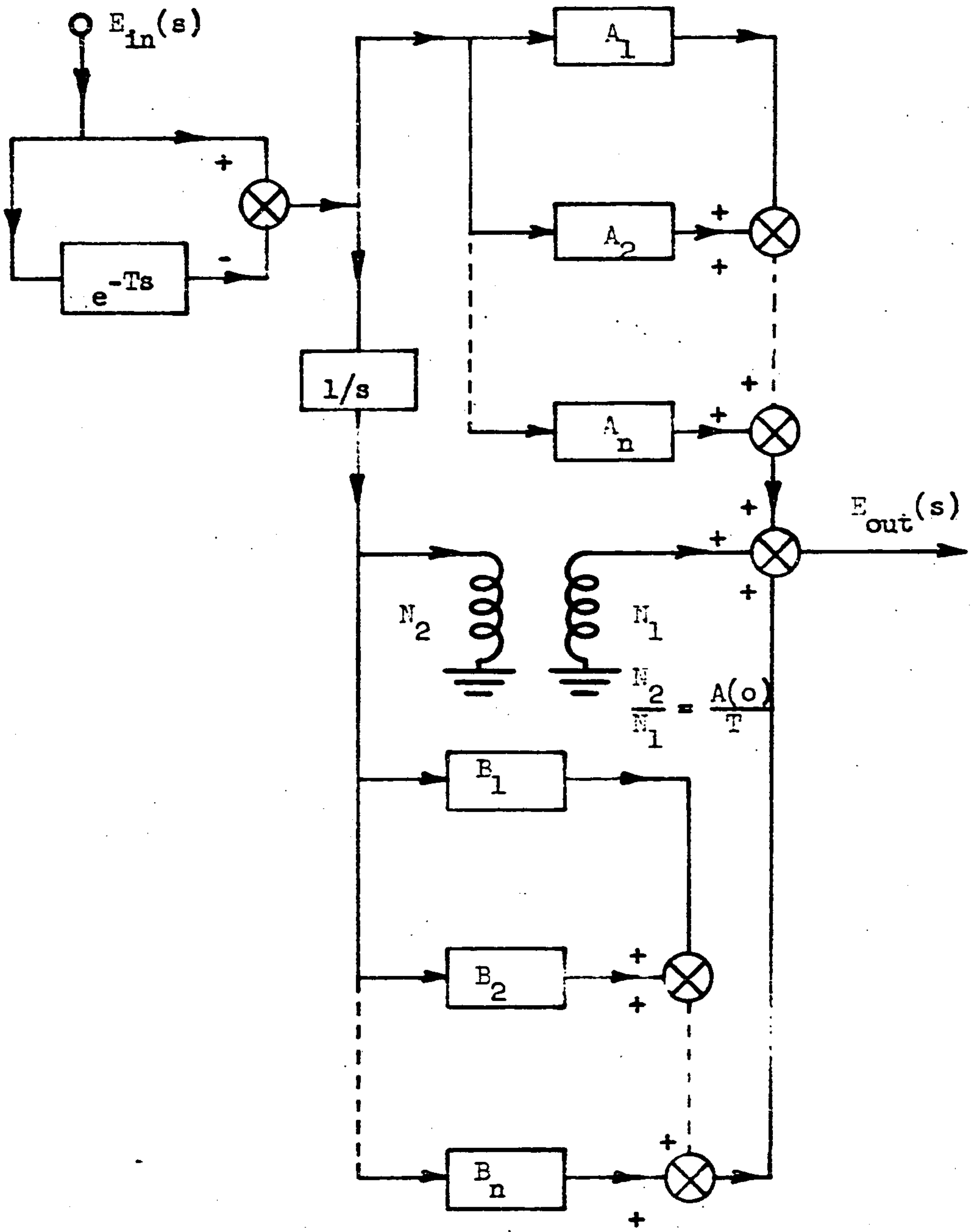


Figure 13. Block diagram of system using a finite number, n, of blocks

IV. MEASUREMENT GUIDE-LINES

A. Selection of Sampling Interval

In the preceding discussion it was shown that the sampling interval, ω_o, was inversely proportional to the duration, T, of the impulsive response, y(t). If T were known, the sampling interval could easily be determined. However, when one begins to take measurements on a particular object, exact knowledge of T is not available. Therefore, the precise sampling interval is unknown and must be determined as a part of the measurement process.

To determine the sampling interval, a certain amount of trial and error is required. Fortunately however, by using a little intuitive reasoning, an approximate measure of T, and therefore ω_o, can be obtained rather quickly by simply examining the size of the object of interest. The reasoning is as follows: If a plane wave were to impinge upon the object, a reflection or return would be initiated at the moment that the wave struck the front of the object. The wave would then traverse the object until it reached the back side and another reflection would occur. The composite return would essentially subside shortly after this second reflection, and thus the duration of the return would be expected to be "roughly" equal to the time that it takes the plane wave to sweep across the object. If the largest path distance from the front to the back of the object is called d, then

$$T \simeq \frac{d}{c}$$

where c is the velocity of the light.

Admittedly this is a rough approximation but it does provide a starting point for the measurement process.

Knowing T approximately, the sampling interval is known approximately, and measurements can be taken throughout the spectral range of interest. As these measurements are taken, noticeable variations will occur in $A(n\omega_o)$ and $\phi(n\omega_o)$. The violence of these fluctuations will also act to confirm or contradict the sampling-interval choice and some cursory adjustment can be made. Once the interval "appears" to be correct on the basis of sweeping time and fluctuations in the measured values, a mathematical check can be made. To perform this check, the sampling-point values are used to predict the values say midway between the sampling points. The prediction function is just the finite-terms version of the sampling function discussed earlier. For example, if the spectral range is $W_1 \leq \omega \leq W_2$, the value of the reflection function at any point, ω', is given by:

$$F_t(\omega') = \sum_{n=\frac{-W_2}{\omega_o}}^{\frac{-W_1}{\omega_o}} F_t(n\omega_o) e^{-j\left[\left(\frac{\omega' T}{2}\right) - n\pi\right]} \frac{\operatorname{Sin}\left[\frac{\omega' T}{2} - n\pi\right]}{\left[\frac{\omega' T}{2} - n\pi\right]}$$

$$+ \sum_{n=\frac{W_1}{\omega_o}}^{\frac{W_2}{\omega_o}} F_t(n\omega_o) e^{-j\left[\frac{\omega' T}{2} - n\pi\right]} \frac{\operatorname{Sin}\left[\frac{\omega' T}{2} - n\pi\right]}{\left[\frac{\omega' T}{2} - n\pi\right]},$$

where W_1 and W_2 are adjusted so as to make W_1/ω_o and W_2/ω_o integer values.

Having performed such a calculation at each of the points which lie midway between the original sampling points, measurements are taken at these same points for comparison. If the comparison is satisfactory the original choice is deemed correct; if not, the sampling interval is chosen to be one half its original size and the process is repeated. Once a satisfactory comparison is reached, the sampling interval is known and the values of the complex reflection function can then be used in the synthesis scheme described earlier. Fortunately, the sampling interval that is used in the measurement scheme need not be exactly equal to the ideal sampling interval; as long as the samples are taken at least as close together as the ideal sample spacing, no information is lost. Of course in the interest of efficiency it is not wise to take too many samples. The method described above will allow the approximate sample spacing to be determined without losing information and without sacrificing efficiency. At worse, the approximate spacing will never differ from the ideal spacing by more than a factor of two, and it will always be less than or equal to the ideal spacing.

To understand why no information is lost in this kind of sampling process, it is only necessary to recognize that using a sampling interval which is slightly less than ideal, corresponds to expanding $y(t)$ in a Fourier series which has a fundamental component with period slightly larger than T. Since $y(t)$ is confined to the interval, $o \leq t \leq T$, nothing is lost by examining $y(t)$ over a slightly larger period.

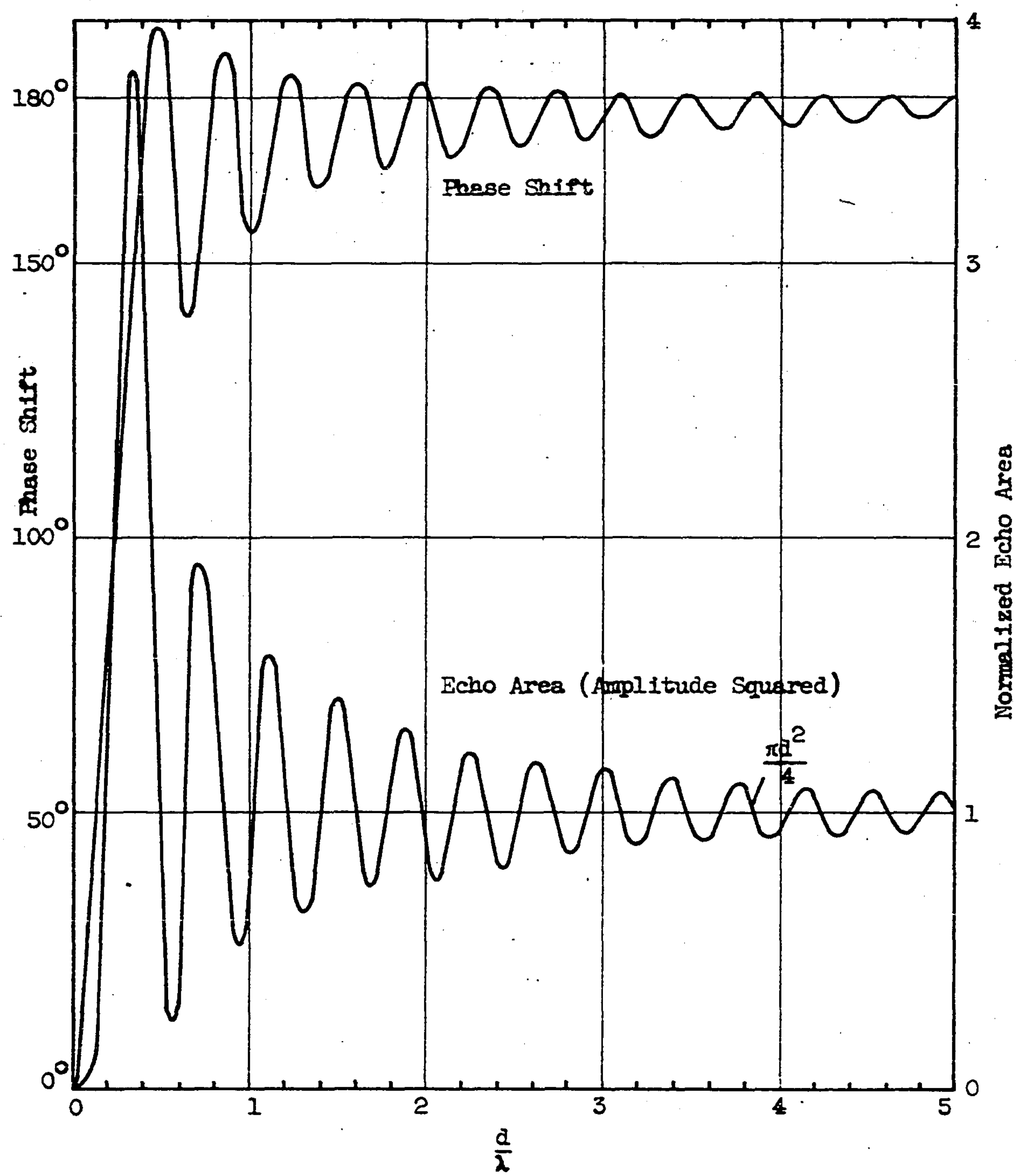
B. Selection of Spectral Interval

In attempting to decide on a spectral interval, $W_1 \leq \omega \leq W_2$, over which to make measurements for any particular object, very little specific information is available. However the Advanced Electronic Systems Group at General Dynamics Convair, San Diego, California (2) has provided a plot of the interesting portion of the frequency spectrum of a conducting sphere. This plot is shown in Figure 14, and it provides a certain amount of guide-line information which may prove helpful in general. Notice that both the phase shift and <u>normalized echo area</u> are plotted as a function of d/λ where d is the diameter of the conducting sphere. Since λ is inversely proportional to frequency, d/λ is proportional to frequency.

The term "echo area" (sometimes called "radar cross-section") has been used historically (5, p. 445) instead of amplitude because in the early studies of radar-target reflection-functions no attempt was made to examine the complex (both amplitude and phase) character of the reflection process. Actually, the echo-area function is just the squared-amplitude function. Squared-amplitude is of course proportional to power. Reflected power is in turn proportional to the object's cross-sectional area for illuminating wavelengths which are much smaller than the physical dimensions of the object. By dividing the echo-area function by the cross-sectional area ($\pi d^2/4$) of the sphere, the so-called normalized echo-area function is obtained. It is this function that is plotted in Figure 14.

The choice of d/λ was made purposely to illustrate three points, i.e., three useful pieces of information: Note that the variations in both phase and normalized echo-area are most violent in the vicinity of $d/\lambda = 1$, i.e., in the region where the illuminating wavelength is nearly equal to the sphere diameter. Notice also that when the wavelength is increased

Figure 14. Phase and amplitude components of the backscatter transfer function for a sphere


Phase Shift
180°
150°
100°
50°
0°
Phase Shift
Echo Area (Amplitude Squared)
$\frac{\pi d^2}{4}$
Normalized Echo Area
0
1
2
3
4
5
$\frac{d}{\lambda}$

to several times the sphere diameter, the variations are much less violent. After an increase of an order of magnitude or two (5, p. 453), the variations are small enough to become negligible and the echo area approaches a constant value equal to the geometrical cross-sectional area of the sphere. This behavior seems reasonable in the physical optics region of wavelengths, since in practice when one views any nominal-sized object with various colors of light the object still appears to have the same reflecting cross-sectional area. On the other extreme, when the illumination wavelengths are an order of magnitude or two larger than the dimensions of the object, one would intuitively expect only an increasing amplitude variation as illumination wavelengths are decreased because the target is so small that it has very little effect on the reflection process. Also the effect would be expected to increase as the size of the target begins to take up more and more of the wavelength of the impinging wave. Such an increase in the low-frequency end of the spectrum is observed in Figure 14.

These arguments are admittedly intuitive and can provide only a very rough insight but at least they do provide a starting point for measurements. That is to say that since the low-frequency behavior seems to be an increasing one for increasing frequency, and since the high-frequency behavior seems to become constant with increasing frequency, one would expect the fluctuating behavior to occur in the mid-frequency range, i.e., where the illumination wavelength is comparable to the dimensions of the object. Because of the relationship that exists between object dimensions and illumination wavelengths, a scaling property is obvious; the larger the object, the larger the wavelength must be in order to lie in the region of fluctuation, and conversely. The region in which the fluctuations occur is deemed the region of greatest interest, and by beginning measurements in this region and working away from it in both directions, the greatest amount of information can be collected in the shortest time.

It should be noted that the intuitive arguments stated above are felt to apply in general but are only demonstrated here by one specific example, namely, that of a conducting sphere. Also, the reasoning was confined to the echo-area behavior rather than to both the phase and echo area behaviors. In the case of the sphere the variational phase behavior seems to be confined to the same interval as the variational echo area behavior. Hopefully this property will be found true in general. However adequate spectral coverages of other interesting objects do not appear to be available in sufficiently broad ranges to provide confirmation. This lack of information, however, adds to the importance of experimental investigation and in part justifies the study presented here.

V. CONCLUSIONS

On the basis of this study it is apparent that complex reflection-functions exist for reflecting objects which are useful in characterising such objects in a manner somewhat equivalent to the representation of electrical networks by complex transfer-functions. Each reflection function can be used with arbitrary inputs to determine corresponding outputs (reflected signals).

Although analytic determination of these functions is quite limited, the functions themselves can be measured by two techniques. One involves the use of unit impulses and the other involves sampling in the frequency domain. Emphasis here has been placed on the latter. This latter method allows the reflection function to be measured in a simple manner and also allows the raw data to be used in a straightforward manner to obtain a synthetic network equivalent of the object's spectral behavior.

Such network equivalents can then be combined in an elaborate array to form a recognition system which can be used to identify unknown objects (at least in an approximate sense) in terms of known objects of elementary shape. For example, by cross-correlating the return from an unknown object with the return from objects of simple elementary shapes such as a sphere, a rod, a cube, a bar, etc., it appears possible to identify the significant characteristics of the unknown object. The system performs the cross-correlations by using matched-filter techniques.

Aside from their use in such a recognition scheme, the reflection functions also provide information which should prove useful in matching modulation spectra of illuminating radars with the spectral character of the objects being observed, or tracked.

Definite guide-lines are available which can be used to expedite the future measurement of interesting reflecting objects in an efficient manner while preserving the informative character of the object. These guide-lines have to do with the selection of spectral ranges of interest and with the selection of proper sampling intervals. Both selections are made on the basis of the object's size (whereas the informative character depends on the object's shape). By adhering to these guide-lines the required information collection is reduced to a straightforward radio illumination and detection problem.

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VIII. APPENDIX A

Although many different limiting techniques can be employed in the definition of the unit impulse, one (1, p. 255) is particularly suitable for Laplace transformation. The definition is made in terms of the difference of two unit-step functions, i.e.,

$$\delta(t) \triangleq \lim_{a \to o} \left[\frac{u(t) - u(t-a)}{a}\right]$$

where $\delta(t)$ is called the unit impulse and $u(t)$ is called the unit step function. The unit step function is defined as

$$u(t) \triangleq \begin{Bmatrix} 1, & t > o \\ o, & t < o \end{Bmatrix} ;$$

thus the delayed unit step function is written as $u(t-a) = \begin{Bmatrix} 1, & t > a \\ o, & t < a \end{Bmatrix}$.

The Laplace transform of $\frac{u(t) - u(t-a)}{a}$ is given by

$$\int_{o^+}^{\infty} \left[\frac{u(t) - u(t-a)}{a}\right] e^{-st}\, dt = \frac{1 - e^{as}}{as} ,$$

Thus $\frac{u(t) - u(t-a)}{a}$ and $\frac{1 - e^{-as}}{as}$ form Laplace transform pairs. By taking the limit of each expression as $a \to o$, the first expression becomes the unit impulse as previously defined, and the second becomes the Laplace transform of the unit impulse:

$$L\left[\delta(t)\right] = \delta(s) = \lim_{a \to o} \left[\frac{1 - e^{-as}}{as}\right] = \lim_{a \to o} \left[\frac{o + se^{-as}}{s}\right],$$

so $\delta(s) = 1$.

Thus the Laplace transform of a unit impulse is seen to be unity. The Fourier transform of the unit impulse previously described is given by:

$$\delta(\omega) = \lim_{a \to o} \int_{-\infty}^{\infty} \left[\frac{u(t) - u(t-a)}{a}\right] e^{-j\omega t} dt \quad \lim_{a \to o} \int_{o}^{a} \frac{e^{-j\omega t}}{a}$$

$$= \lim_{a \to o} \left[\frac{1}{-j\omega} \; \frac{e^{-j\omega t}}{a} \; \Big|_{o}^{a}\right]$$

$$= \lim_{a \to o} \left[\frac{e^{-j\omega a} - 1}{-j\omega a}\right]$$

which tends to the indeterminate form, $\frac{o}{o}$. Application of L'Hospital's Rule gives

$$\delta(\omega) = \lim_{a \to o} \left[\frac{-j\omega \, e^{-j\omega a} - o}{-j\omega}\right] = \lim_{a \to o} \left[e^{-j\omega a}\right]$$

$$= 1.$$

Thus the Fourier transform of the unit impulse is also unity.

IX. APPENDIX B

Let s(t) be a real valued function of the real valued variable t. By defining the function S(ω) as

$$S(\omega) = \int_{-\infty}^{\infty} s(t)e^{-j\omega t}dt = \int_{-\infty}^{+\infty} s(t)\mathrm{Cos}(\omega_c t)dt - j\int_{-\infty}^{+\infty} s(t)\mathrm{Sin}\ \omega_c t dt,$$

it is clear that S(-ω) is given by

$$S(-\omega) = \int_{-\infty}^{\infty} s(t)e^{-j\omega t}dt = \int_{-\infty}^{+\infty} s(t)\mathrm{Cos}(\omega_c t)dt + j\int_{-\infty}^{+\infty} s(t)\mathrm{Sin}(\omega_c t)dt.$$

Now $$|S(\omega)| = \left\{\left[\int_{-\infty}^{+\infty} s(t)\mathrm{Cos}(\omega_c t)dt\right]^2 + \left[\int_{-\infty}^{+\infty} s(t)\mathrm{Sin}(\omega_c t)dt\right]^2\right\}^{\frac{1}{2}},$$

and, also,

$$|S(-\omega)| = \left\{\left[\int_{-\infty}^{+} s(t)\mathrm{Cos}(\omega_c t)dt\right]^2 + \left[\int_{-\infty}^{+} s(t)\mathrm{Sin}(\omega_c t)dt\right]^2\right\}^{\frac{1}{2}}.$$

Thus $|S(-\omega)| = |S(\omega)|$, so $|S(\omega)|$ is an even function.

Furthermore,

$$\phi_s(\omega) = \tan^{-1}\left\{-\left[\int_{-\infty}^{+\infty} s(t)\mathrm{Sin}(\omega_c t)dt\right]\Big/\left[\int_{-\infty}^{+\infty} s(t)\mathrm{Cos}(\omega_c t)dt\right]\right\},$$

and

$$\phi_s(-\omega) = \tan^{-1}\left\{+\left[\int_{-\infty}^{+\infty} s(t)\mathrm{Sin}(\omega_c t)dt\right]\Big/\left[\int_{-\infty}^{+\infty} s(t)\mathrm{Cos}(\omega_c t)dt\right]\right\}$$

$$= - \tan^{-1}\left\{ -\left[\int_{-\infty}^{+\infty} s(t)\text{Sin}(\omega_c t)dt\right] \Big/ \left[\int_{-\infty}^{+\infty} s(t)\text{Cos}(\omega_c t)dt\right]\right\} .$$

Thus $\phi_s(-\omega) = -\phi_s(\omega)$, so $\phi_s(\omega)$ is an odd function.

X. APPENDIX C

The easiest way to show that the functions

$$f(t) = \text{Cos}(\omega_c t) \text{ and } F(\omega) = \pi\,[\delta(\omega + \omega_c) + \delta(\omega - \omega_c)]$$

form a Fourier transform pair is to take the inverse Fourier transform of $F(\omega)$.

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)e^{+j\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \pi\,[\delta(\omega+\omega_c) + \delta(\omega-\omega_c)]\;\; e^{-j\omega t}d\omega$$

$$= \frac{1}{2}\;[e^{-j\omega_c t} + e^{+j\omega_c t}\;]$$

$$= \text{Cos}\;\omega_c t.$$

Although it is also possible to proceed in the forward direction, extensive argument is needed. The effort, although instructive, is not felt to be justifiable in this discussion.

XI. APPENDIX D

Consider any function, s(t), which is Fourier transformable. If

$$F[s(t)] = \int_{-\infty}^{+\infty} e^{-j\omega t} s(t)dt = S(\omega),$$

it is interesting to find F[s(-t)] in terms of S(ω).

To do this, let $-t = \tau$. Then $t = -\tau$ and $dt = - d\tau$. Also, when $t = - \infty$, $\tau = + \infty$. Furthermore, when $t = + \infty$, $\tau = - \infty$. Thus, the Fourier transform F[s(-t)], which is normally written as

$$\int_{-\infty}^{\infty} e^{-j\omega t} s(-t)dt,$$

can now be written as

$$\int_{+\infty}^{-\infty} e^{j\omega\tau} s(\tau)(-d\tau)$$

$$= \int_{-\infty}^{+\infty} e^{j\omega\tau} s(\tau)d\tau.$$

By making another substitution, i.e., $\omega = -W$, F[s(-t)] becomes

$$\int_{-\infty}^{+\infty} e^{-jW\tau} s(\tau)d\tau,$$

which is just the Fourier transform, S(W), of s(τ). Thus,

$$F[s(-t)] = S(W) = s(-\omega).$$

Furthermore, since

$$S(\omega) = \int_{-\infty}^{+\infty} s(t)e^{-j\omega t}dt = \int_{-\infty}^{+\infty} s(t)\mathrm{Cos}(\omega t)dt - j\int_{-\infty}^{+\infty} s(t)\mathrm{Sin}(\omega t)dt$$

and

$$S(-\omega) = \int_{-\infty}^{+\infty} s(t)e^{+j\omega t} = \int_{-\infty}^{+\infty} s(t)\mathrm{Cos}(\omega t)dt + j\int_{-\infty}^{+\infty} s(t)\mathrm{Sin}(\omega t)dt,$$

it is clear that $S(\omega)$ and $S(-\omega)$ are complex conjugates, i.e., $S(-\omega) = S^*(\omega)$. Therefore, since $F[s(-t)]$ is equal to $S(-\omega)$, it is also equal to $S^*(\omega)$. Clearly, then, $F[s(-t)]$ and $S^*(\omega)$ form a Fourier transform pair. Moreover, if one uses the notation $s^*(t)$ for $F^{-1}[S^*(\omega)]$, it becomes clear that

$$s^*(t) = s(-t).$$